

EXPERIMENTAL AND NUMERICAL STUDY OF EFFECTIVE THERMAL CONDUCTIVITY MODELS LIMITATIONS AND SELECTION OF POROUS MEDIA: REVIEW

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ABSTRACT

Efficient thermal conductivity (ETC) within porous material is an important aspect of numerous engineering endeavors such as geothermal energy systems, thermal insulation and material science. The correct modeling of ETC is critical to the heat transfer awareness mechanism of porous structures, which may possess complicated pore stratagems. This literature review examines the drawbacks and the criteria used to select different thermal conductivity models to use on porous materials. It looks at the classical models, including Zehner-Schlunder and Maxwell and more sophisticated models, including fractal models, percolation theory and Lattice Boltzmann simulations. The performance of every model is compared with other physical conditions; however, the attention is paid to the issues of pore size distribution, fluid saturation, or geometry. The review points to the necessity of choosing the models depending on the properties of the porous medium, i.e., metal foams, granular substances, or sintered structures. The major findings indicate that at the small-scale level, traditional models can be useful in simple structures but cannot be relevant in more complicated geometries including the ones that are found in fractured rocks or nanoporous materials. Besides, the review reiterates the significance of the fact that temperature and fluid saturation effects should be considered as in many models they tend to be disregarded. The experiments are also essential in order to fine-tune the predictions of these models. Finally, it is possible to infer that the current research proposes that additional research is needed to make ETC models more accurate through the inclusion of more realistic geometries of pores, temperature dependencies and fluid interactions. It additionally requires the creation of hybrid models that integrate theory with numerical models to deal with the details of the porous materials in the world.

Keywords: Effective thermal conductivity, Porous media model, Numerical and experimental review, Model selection and limitations, Multiscale porous materials.

NOMENCLATURE

K Thermal conductivity
g Grain phase

S_w Wetting phase saturation
 D_{fs} The fractal dimension of grains

Subscripts

e Effective

n_w Non-wetting phase

types of materials. Choosing the right model for a particular porous structure is a big challenge as no model can accurately represent all types of porous media.

Many approaches have been employed in the literature regarding this task, ranging from classic ones like the Zehner-Schlunder and Maxwell models to more intricate modelling fractals and percolation. Recently, researchers have proposed lattice Boltzmann simulations and hybrid models to better understand heat conduction in porous structures (Shen & Rose, 2024; Qin et al., 2020). The goal of these studies is to better comprehend how pore shapes and sizes, heterogeneous materials, and the interaction between solid and fluid contribute to thermal conductivity.

Despite the progress, several challenges remain the thermal conductivity of porous materials is highly dependent on pore structure, so a universal model that describes it and estimates heat conduction through the solid frame is essential. A significant challenge in this area is posed by the wide diversity of pore structure, from simple spherical pores to more complicated ones that are fractal-like (Shen et al., 2024; Xu et al., 2021). Some other more factors like temperature dependency, saturation levels, the interactions between solid and fluid phases lead to differences in predicted and measured thermal conductivities. Research has proven to validate and refine numerical methods, but disagreement continues to exist, especially for non-ideal pores like metal foams or fractured rocks (Florez et al., 2013; Wang et al., 2021).

This study reviews existing models for predicting effective thermal conductivity, highlighting the limitations of these models, and provides recommendations for selecting appropriate predictive schemes for different types of porous materials. This paper will address difficulties of modeling thermal conductivities of porous media with an emphasis on recent advances in numerical and experimental studies. It will also indicate some future research perspectives derived from this work. The review will also address the recent understanding of the effects on effective thermal conductivity by geometry, saturation, and material property as well as remaining research challenges.

THEORETICAL MODELS FOR PREDICTING EFFECTIVE THERMAL CONDUCTIVITY

The theoretical models are the simplest ones that comprise the estimation of the ETC of porous materials and are the subject of this section. The main discussed models shall be Zehner-Schlunder and Maxwell models frequently employed on the granular and porous media. The assumptions of each of the models based on the homogeneity of the pore structure and interaction between the solid and fluid phase will be tested.

- **Research Methods and Analysis:** The theoretical models make simplifying assumptions concerning the geometry of the pores (e.g., cylindrical or spherical) and homogeneous material. Investigations made because of such models usually impose idealistic assumptions, like constant thermal conductivity or an example of fluid saturation, which might not be true of real materials.
- **Evaluation and Comparison:** simple porous structures are modeled well but highly irregular or complex geometry of the pore is not well modeled by these models. The Maxwell model can, as an example, give too high or too low values of the ETC of materials with important pore irregularity or fractality such as in natural materials such as fractured rock or materials sintered.

- Implications and Limitations: These models are very easy to compute so that results can be obtained quickly, although they do not take into account the complexity of heterogeneous and irregular pore networks. They need adaptations or modifications in the porous media of complex structure, which causes the consideration of more sophisticated models.

To understand the applicability and limitations of different thermal conductivity models for porous media, it is necessary to classify them according to the properties of the porous media to which they apply or are specifically aimed. Many controversial issues can be resolved by sorting thermal conductivity models based on type of porous media and their assumptions on structure, saturation and pore geometry. This categorization will aid researchers in choosing the right model based on the kind of porous media being studied.

Models for granular media

Granular materials usually contain randomly packed particles and have simple pore structure. According to this image, different models can be used depending on the type of porous media in consideration.

- a) The Zehner-Schlunder Model (Hsu et al., 1994; Takatsu et al., 2016) This model is largely used for granular media particularly in cases where the material displays a more homogenous distribution of pores. It takes into consideration the stagnant fluid thermal conductivity as well as the particle packing structure's influence. As in the following equations:

$$\frac{k_e}{k_f} = 1 - \sqrt{(1 - \phi)} + \frac{2\sqrt{(1-\phi)}}{1-\lambda B} \left(\frac{(1-\lambda)B}{(1-\lambda B)^2} \ln \frac{1}{\lambda B} - \frac{B+1}{2} - \frac{B-1}{1-\lambda B} \right) \quad (1)$$

$$B = C \left(\frac{1-\phi}{\phi} \right)^m, \text{ and } \lambda = k_f/k_s \quad (2)$$

Where $C = 1.25$, $m = 10/9$, $\phi = 0.42$

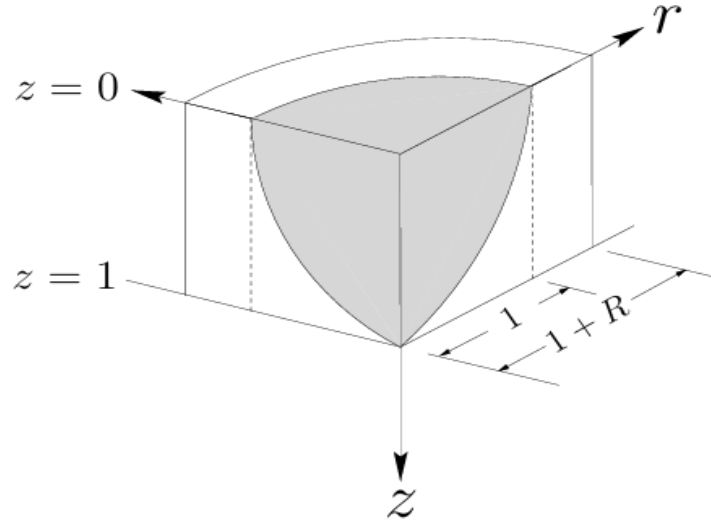


Fig .1. Unit cell used by Zehner and Schlunder (Aichlmayr and Kulacki, 2005).

- b) Fractal models help making an improved prediction on complex pore networks with self-similarities for granular media. (Qin et al., 2019) These models assist in estimating ETC using the fractal dimension of the pore geometry.

A fractal thermal conductivity model for saturated porous media as in the following equations:

$$k_e = k_f + \frac{(D_{fs}-1)^3 \phi}{D_{fs}^2(3-D_{fs})} \frac{3k_f(k_s-k_f)}{k_s+2k_f} \quad (3)$$

A fractal thermal conductivity model for unsaturated porous media as in the following equations:

$$k_e = [\alpha k_w + (1 - \alpha)k_a] + \beta \frac{[\alpha k_w + (1-\alpha)k_a]\{k_s - [\alpha k_w + (1-\alpha)k_a]\}}{k_s + 2[\alpha k_w + (1-\alpha)k_a]} \quad (4)$$

k_w and k_a are the thermal conductivity of the liquid and the air phases where:

$$\alpha = \frac{D_{fw} + D_{Tw} - 1}{D_{fw}} S_w^{1 + \frac{D_{Tw} - 1}{D_E - D_{fw}}}, \beta = \frac{3(D_{fs} - 1)^3 \phi}{D_{fs}^2(3 - D_{fs})} \quad (5)$$

fractal dimensions of the granular matrix and the liquid phases (D_{fs}, D_{fw}), the tortuosity fractal dimension of the liquid phase (D_{Tw}), liquid saturation (S_w).

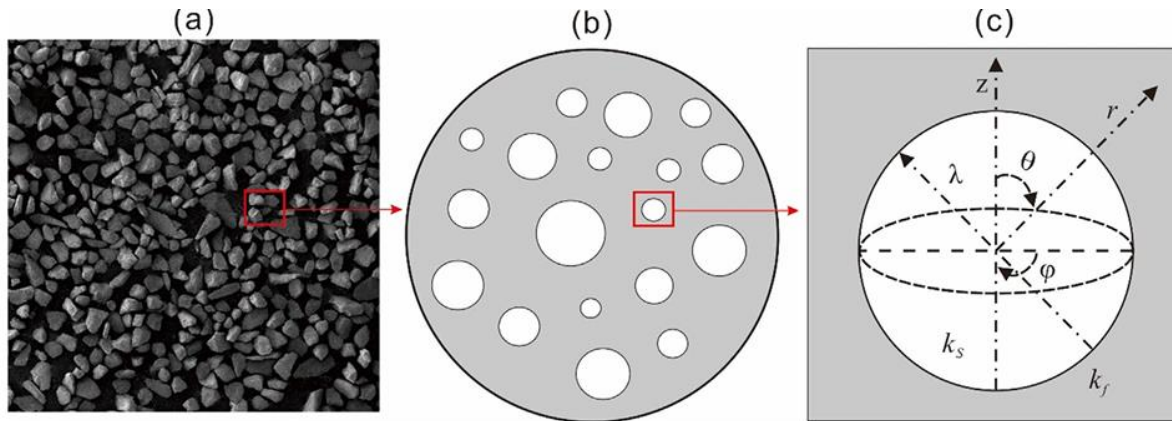


Fig .2. Schematic illustrations of the granular material. (a)The micro-CT image of a granular rock sample. (b)The unit cell of the effective media. (c)A single particle contained within a continuous medium (Qin et al., 2019).

- c) The modified Maxwell models were developed by (Behrang et al., 2016). This type of model can be modified to accurately study not only granular media but nanoporous structures as well when the geometries are irregular in shape and solid and fluid phases are mixed. As in the following equation:

$$Ke = Knw Kw * (2\emptyset(Kw(1 + \emptyset(-1 + Sw))Sw + Knw(-1 + Sw)(-1 + \emptyset Sw)) + Kg(3 + \emptyset(-2 - \emptyset(-1 + Sw)Sw))) / (Kg Kw \emptyset(-1 + Sw)(-1 + \emptyset Sw) + Knw(kg \emptyset(1 + \emptyset(-1 + Sw))Sw + Kw(3 + \emptyset(-1 + \emptyset(-1 + Sw)Sw))))$$

(6)

Using these models isn't really a good option if the geometry of the pore structure is complicated to a great extent, for e.g. in sand, soil etc.

Foam materials model (metal foam, polymer foam, etc.)

Foams are materials with enormous porosity and with regular, interconnected pores. These materials have high porosity and are often used as thermal insulators. The properties of foams like the skeleton of the material, pore size distribution, and porosity affect the thermal conductivity of foams.

- a) The modified Maxwell model (Zhu, 2019): The effective thermal conductivity of foam materials can be predicted effectively when the solid phase and the gas phase in the pores are accounted for.

The dimension less effective thermal conductivity of a porous medium based on the conceptual cell model as in the equation bellow:

$$\frac{k_e}{k_f} = \frac{\left(\frac{R}{R_1}\right)^3 \left\{ (k_s/k_f + 2) \left[\left(\frac{R_1}{R}\right)^3 - 1 \right] + 6(k_s/k_f - 1) \ln\left(\frac{R_1}{R}\right) + 3(k_s/k_f) \right\}}{k_s/k_f + 2 - (k_s/k_f - 1) \left(\frac{R}{R_1}\right)^3} \quad (7)$$

- b) Effective Medium Theory, (Gong et al., 2013) For modeling the heat transfer in foams, the material is considered as an effective medium comprising solids and fluid components.

$$(1 - \varepsilon) \frac{k_s - k_e}{k_s + 2k_m} + \varepsilon \frac{k_a - k_e}{k_a + 2k_m} = 0 \quad (8)$$

When $K_m = K_e$, is the EMT model. Also, when $K_m = K_s$ and K_a , is the Maxwell–Eucken1and2, but when $K_m = 0$ is the Series model, and when $km = 1$ is the Parallel model.

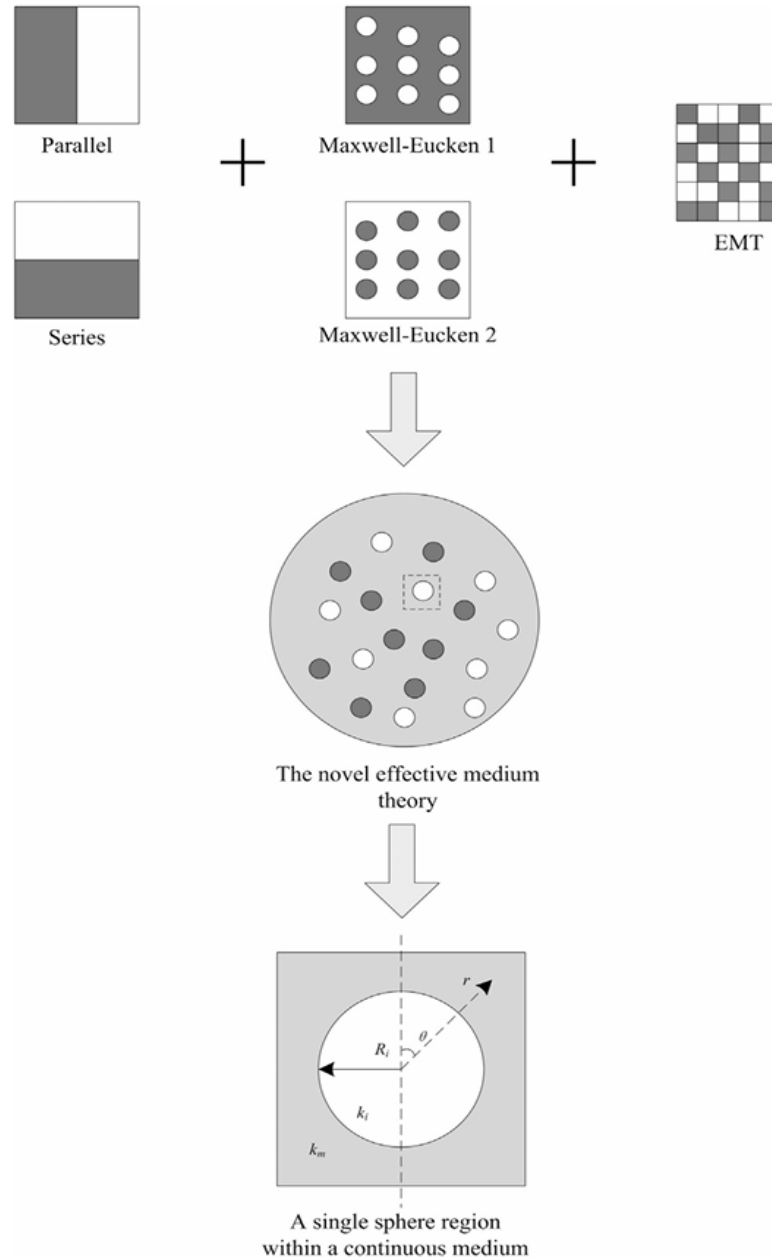


Fig.3. the novel effective medium theory (Gong et al., 2013).

- c) The porous medium approach known as (Wang, 2021) is used for predicting thermal conductivity of the foam material. A specific kind of foam material is used here which is metal foam specifically with high connectivity. The general correlation is also proposed as:

$$\frac{k_{\text{eff}}}{k_s} = 0.6899\varepsilon^2 + \left(\frac{k_f}{k_s} - 1.6899\right)\varepsilon + 1 \quad (9)$$

Where ε is the porosity.

These models are suitable for porous foams used in insulation, soundproofing, or heat exchange applications, where the pore structure is usually uniform but interconnected.

Models for fractured media (rocks, ceramics etc.)

Fractured porous material has a complex pore network, which is most of the time quite irregular. Similarly, its thermal properties may be heterogeneous. Thermal conductivity in fractured materials depends on the fractures themselves and the surrounding solid. Thus, the solid and fluid phases will need to be parameterized.

- a) According to Xu et al. (2021) and Shen et al. (2024), Percolation theory refers to fractured media where pore connectivity plays a large role that determines conductivity. It models the heat conductivity by taking the point, the solid and fluid phases start connecting, known as the percolation threshold. According to the following equation:

$$\frac{T_{eff}}{T_m} = \left[\Omega_T + \sqrt{\Omega_T^2 + \frac{\phi_c}{1-\phi_c} \left(\frac{T_p}{T_m}\right)^{1/t}} \right]^t ; \Omega_T = \frac{1}{2(1-\phi_c)} \left[1 - \phi_c - \phi + (\phi - \phi_c) \left(\frac{T_p}{T_m}\right)^{1/t} \right] \quad (10)$$

Where the effective thermal conductivity T_{eff} of a two-phase material, percolation threshold ϕ_c , The thermal conductivity spheroidal inclusions is T_p and the thermal conductivity of homogeneous matrix is T_m also t is again a transport-percolation exponent. It has been validated that t is a non-universal value greater/less than or equal to 2.

- b) Fractal-oriented models help to characterize the irregularity and complexity of the fracture networks and yield more accurate predictions of the thermal conductivity of materials with highly heterogeneous pore structures (Qin et al., 2019; Shen et al., 2020). The ETC of unsaturated porous media represented by the following equation (Shen et al., 2020):

$$\begin{aligned} k_e = & k_g \frac{(2-D_f)^{(D_T+1)/2}}{D_f^{(D_T-1)/2} (1+D_T-D_f)} \left(\frac{\phi}{1-\phi}\right)^{(D_T+1)/2} \\ & \times \left\{ 1 - [(1-\phi)S_w + \phi]^{\frac{1+D_T-D_f}{2-D_f}} \right\} \\ & + k_w \frac{(2-D_f)^{(D_T+1)/2}}{D_f^{(D_T-1)/2} (1+D_T-D_f)} \left(\frac{\phi}{1-\phi}\right)^{(D_T+1)/2} \\ & \times \left\{ [(1-\phi)S_w + \phi]^{\frac{1+D_T-D_f}{2-D_f}} - \phi^{\frac{1+D_T-D_f}{2-D_f}} \right\} + k_s(1-\phi) \end{aligned} \quad (11)$$

Where the tortuosity fractal dimension D_T , the fractal dimensions for pore D_f , The wetting fluid saturation S_w ,

- c) There are hybrid models (Shen and Rose, 2024) that combine percolation theory with classical heat transfer models. Like the Zehner-Schlunder model. These can be used to better predict ETC in fractured rocks and similar materials.

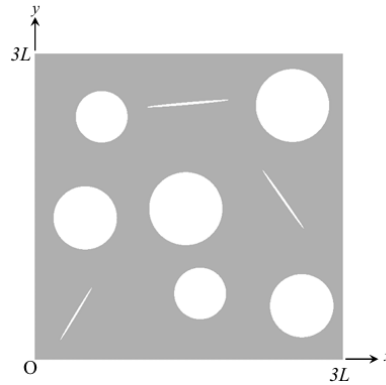


Fig.4. Representative random pore/crack model containing six pores and three cracks in a $3L \times 3L$ domain ($D_c = 0.5D_p$), (Shen and Rose, 2024).

These models can be applied to fractured rocks or other materials that experience crack formation. They can also be utilized for ceramics.

Models for nanoporous and nanostructured materials

Nanoporous materials contain micro-structures with the smallest pore sizes (usually at the nm scale). Such materials create unique challenges for ETC modelling due to the very high surface to volume ratio as well as the influence of molecular transport mechanisms.

- a) The Modified Boltzmann Transport Equation to allow a Boltzmann Distribution of the phonon states and are very useful for Nanoporous materials having effects all over the surface and the phonon scattering mechanism highly dominant.

The Boltzmann Transport Equation can be modified to effectively model nanoscale heat transfer, as shown by (Yun et al.,2022).

- b) Molecular Dynamics Simulations are often used to model heat transfer at nanoscale level for which intermolecular and intramolecular interactions become significant (Qin et al., 2020). These simulations offer information regarding how nanoparticles or nanoporous materials behave.

The discretized form of the double evolution equation with MRT collision model and for the velocity field and temperature field, with a BGK collision model is expressed as (d'Humières et al., Yang et al.):

$$\begin{aligned} f_i(\mathbf{x} + \mathbf{e}_i \delta t, t + \delta t) - f_i(\mathbf{x}, t) &= -\mathbf{M}^{-1} \mathbf{S}[(m_j(\mathbf{x}, t) - m_i^{eq}(\mathbf{x}, t))] \\ g_i(\mathbf{x} + \mathbf{e}_i \delta t, t + \delta t) - g_i(\mathbf{x}, t) &= -\frac{1}{\tau_g} [g_i(\mathbf{x}, t) - g_i^{eq}(\mathbf{x}, t)] \end{aligned} \quad (12)$$

where f_i is the particle density distribution function, \mathbf{M} is the transformation matrix, $m_i = \mathbf{M} \cdot f_i$ is the velocity moments, $m_i^{eq} = \mathbf{M} \cdot f_i^{eq}$ is the equilibrium moments, \mathbf{S} is relaxation matrix, g_i is the internal energy distribution function, g_i^{eq} equilibrium internal energy distribution function, and τ_g is the internal energy relaxation parameter related to the thermal diffusivity.

- c) Fractal models (Qin et al., 2019; Shen et al., 2020) are proposed for nanoporous materials where the pores are of irregular shapes and these models can augment the heat transfer mechanisms by assessing the nanoporous network at various scales. As illustrated in the equation (11).

Application of models: These models apply to the materials such as aerogels, nanoporous ceramics, and carbon nanotube-based materials, etc., where nanoscale properties greatly influence the thermal conductivity.

Models for saturated and unsaturated porous media

The effective thermal conductivity of the porous media depends on whether the material is saturated with a fluid (liquid or gas) or fluid saturation is partial (unsaturated). The classification influences the heat transfer process which takes place in the fluid filled pores.

- a) Saturated Media Models (Zhu, 2019; Qin et al., 2019) models for saturated porous media usually assume complete saturation where heat transfer occurs both in the solids and the fluids. They change these models to study the effect created by the type of fluid (water, air) in the pores. As illustrated in the equations (3) and (7).
- b) Models developed for unsaturated media, including fractal geometry and liquid-gas interface models, predict the ETC for all fluid saturations <100%. (Shen et al., 2020; Qin et al., 2019) These models think about how the liquid and gas phases interact in the pore network. As illustrated in the equations (4) and (11).

These models are used in geothermal engineering, soil science and also in materials where porosity varies from fully saturated to unsaturated conditions, like the rocks or foams used in thermal insulation.

Models for low and high porosity media

The effective thermal conductivity of a material is greatly altered by porosity, especially in the case of low/ high porosities. Heat conduction in low porosity materials is generally solid phase dominated while it is fluid dominated for high porosity materials

- a) Low porosity coals have typically focused only on heat conduction through the solid only (Hsu et al. 1994, Bi et al. 2024). These models are often simple since the void space of low-porosity coal does not contribute much to the overall thermal conductivity. As illustrated in the equation (1).

- b) High porosity materials with respect the void space, require complex models that take into account the tortuosity and gas phase heat conduction (Shen et al., 2024; Ghanbarian & Daigle, 2015). The effective thermal conductivity of the pore-containing structure is

$$K_{\text{eff}} = \frac{JL}{T_1 - T_0}, \quad (13)$$

where J is the total reaction heat flux obtained from the simulation.

These models have application in a wide range of uses such as insulation (high porosity) to structural materials (low porosity) where effect of porosity on thermal conductivity is important.

MATHEMATICAL APPROACHES TO THE MODEL OF EFFECTIVE THERMAL CONDUCTIVITY

Generally, numerical methods (lattice Boltzmann simulations, finite element analysis (FEA) etc.) are becoming more popular in an effort to address the shortcomings of theoretical models. Complex pore network and fluid solid interaction effects can be better modeled by these methods.

- **Research Methods and Analysis:** No details on the research methods and analysis are given because numerical studies frequently aim at the pore scale and are interested in realistic description of the geometry and flow trajectories. Given that the lattice Boltzmann modes can be used to simulate the flow of fluids and distributions of temperatures in the pores of porous substances, it is possible to elaborate on the mechanisms of their analysis. Finite element models, however, provide solution to a heat conduction problem, which is obtained numerically to determine the thermal conductivity of any given material, depending on its shape.
- **Evaluation and Comparison:** The methods are more precise in prediction over theoretical models particularly when geometries are intricate and heterogeneous materials. They however have a high computational cost, especially where the large-scale systems or materials having a fine-scale structure are being utilized. The lattice Boltzmann simulation example can be given, which is quite precise at a large scale but expensive.
- **Implications and Limitations:** These numerical techniques can not be used in practice because of the large needs of computational resources, when it is necessary to use high precision in predictions of ETC. Also, the accuracy of these methods is determined by the accuracy of input parameters artificially like pore geometry and fluid properties which can be significantly different in the real world.

The existence of heterogeneity and geometrical complexity of porous media introduces much difficulty of uncertainty in ETA behavior of porous media. When applying many of the classical theoretical models to ETC, the pore structure is assumed to be homogeneous. But, 'real' porous materials are almost never as simple and uniform with respect to pore size, shape, distribution, and presence. Such heterogeneity and the geometrical complexity in their pores prevent prediction of

thermal conductivities. The next will show how they affect ETC models, the problems they arise and the need to better models around all of these factors.

Heterogeneity in porous media

Heterogeneity in porous media means that the properties of the medium change in space and time. Variations occur in the size, shape, distribution, and composition of pores and the connections and entrapment of the pores. The heat transfer has greatly governed by such heterogeneous properties because the conduction of heat will rely on the architecture and connectivity of the pore network. When heat transfer is homogeneous or isotropic, heat will travel through a material in a predictable way. In contrast, when heat conduction occurs in heterogeneous media, the heat conduction will be more irregular and is very hard to model.

- a) Materials with larger pore sizes tend to follow a complex heat transfer process, often exhibiting bimodal or multimodal pore distributions. While larger pores facilitate better heat transport, smaller pores introduce thermal resistance due to a smaller effective area in contact with solids. According to Pastor Molines (2025), sintered and metal foams with varying pore sizes can lead to significant discrepancies between the predicted and actual thermal conductivity of porous media, as these variations are often overlooked by conventional models."
- b) The irregular shapes and non-uniform distribution of pores are commonly observed in rocks and soil, which are natural porous materials. Many classic models assume the pores are spherical or cylindrical but the real pore structure may be more complex like irregular cavities, fractures or networks of connected pores. Because the path becomes more complex, the heat through a medium is affected; the same is true for the fluid flow. This phenomenon is especially significant in fractured rocks, where cracks and voids create heterogeneity in the pore structure, affecting the prediction of thermal conductivity (Xu et al., 2021). As illustrated in the equation (10).

Geometrical complexity and tortuosity

The heat conduction behavior in a porous medium is determined directly by the geometrical complexity of the pores and their connectivity. When media have complex geometries, such as those with a high degree of fractality or irregular pore connectivity, the heat flow paths are not simple. Rather, heat moves through a labyrinth of complex paths where it struggles at every turn. Tortuosity is when heat can't flow through a straight line and can happen due to the pore structure (complexity of a porous medium). It plays a major role in effective thermal conductivity.

- a) The shape of the pore structure affects thermal conductivity. It shows that, instead of molecules moving along a straight line, the heat molecules are having to take a longer route through the pores. As tortuosity increases, heat transfer efficiency decreases. This effect is especially notable if materials are coarse with irregular pore networks, like those that arise from fractures in such material. A model that accounts for tortuosity yielded a more

accurate estimation of ETC than the model assuming a straight-line heat flow. Tortuosity is particularly substantial in fractured media (Shen et al., 2024).

- b) Fractal geometry, Fractals are self-similar structures that can be power-law. Fractal models of heat transfer have been found to be effective for material with a network of pores possessing a power-law behavior at different scales. (Qin and others 2019) assessed how fractal models account for the complex porous media that improve estimation of effective thermal conductivity. This is especially true for materials with a highly irregular pore shape.

Models struggling with heterogeneous and complex geometries

Traditional models like the Maxwell model, effective medium theory, Zehner-Schlunder model, etc., often assume a uniform pore structure or oversimplify the geometry by using spherical or cylindrical pores. These models function well for rather simple and homogeneous structures. For more complex ones (such as those found in nature), they collapse. These models basically fail to predict the impact of variation in pore shape, irregular pore distribution, fracture intensity and crack presence in the material. Thus, these models may either over-predict or under-predict the effective thermal conductivity.

- a) The Maxwell model and effective medium model assumptions include uniformly distributed spherical pores. The models perform adequately for foams and some granular materials Askari et al., (2015), but not for cases with strongly irregular or non-spherical pores [sintered materials (Florez et al., 2013), fractured media]. These models do not take into account the effects of intricate pore connections and the twisted routes that heat must follow. The Maxwell–Eucken and EMT models were also modified by Hamilton and Kirkpatrick (Wang et al., 2006), as described by the following equations (Gong et al., 2013):

$$k_e = k_1 \frac{(f-1)k_1+k_2-(f-1)(k_1-k_2)v_2}{(f-1)k_1+k_2+(k_1-k_2)v_2} \quad (14)$$

$$(1 - v_2) \frac{k_1 - k_e}{k_1 + (f/2 - 1)k_e} + v_2 \frac{k_2 - k_e}{k_2 + (f/2 - 1)k_e} = 0 \quad (15)$$

Where k and v are thermal conductivity and volume fraction, and subscripts of e , 1 and 2 represent the two-component material system, components 1 and 2, respectively, And (f) the weighting parameter between (0 and 1).

- b) Percolation Models: On the other hand, percolation theory is more capable of modelling materials with complex and disordered pore structures. (Xu et al., 2021). As illustrated in the equation (10). Percolation models take into account the connection of pores, which provides a critical threshold for percolation with regard to heat being able to conduct. But these models still cannot be used for materials with very high heterogeneity or extreme morphological complexity. Although they give a better representation of heat transfer in

such media, they still rely on simplified assumptions about pore size distribution and connectivity.

Addressing heterogeneity and geometrical complexity in modern models

Recent developments in modeling approaches have been made to deal with limitations due to heterogeneity and geometrical complexity. Evolving models are added with more pore structure detail representation and variation representation of material properties with scale.

- a) Modern models use a combination of theories on multiple scales to capture the pore-scale characteristics as well as the bulk material behavior of the system. These models can take into account the variation at various length scales, from the pore size to the material size. Hybrid models that incorporate Lattice Boltzmann methods (Zhou, F., & Cheng, G. 2014) as illustrated in the equation (12), with percolation theory can model the thermal behavior in multi-scale geometries better (Shen & Rose, 2024) By solving the heat conduction at micro and macro scales, these models can better predict the effective thermal conductivity of heterogeneous materials.
- b) Fractal models as per Qin et al. (2019) has been widely applied on materials with irregular pore geometries. These models also apply self-similarity of pore structure to enable predictions of effective thermal conductivity in materials having pore sizes following a power law. The effects of pore size distribution and connectivity patterns can be accounted in the model to improve the prediction.

EXPERIMENTAL APPROACHES FOR VALIDATING THERMAL CONDUCTIVITY MODELS

The validation of theoretical and numerical models is done with experimental validation and verification. This part will discuss different experimental processes of evaluating the effective thermal conductivity of porous material between steady state (e.g. guarded hot plate) and transient (e.g. laser flash and transient line-source methods).

- **Research Methods and Analysis:** The thermal conductivity of a material under controlled conditions, with known methods of test, is usually measured. The comparison of the experimental data against the model predictions to detect discrepancies with a view of correcting the models is then established. It is common to have varying conditions in which experiments are carried out in studies including temperature range, fluid saturation and sample geometry.
- **Comparison and evaluation:** Experimental validation helps give useful information about the real behaviour of porous materials. As an example, in the works by Florez et al. (2013), ETC values were predicted and measured as not equal with the measured values of sintered porous materials as the pore structure is irregular. These investigations are the cause of the necessity of models that consider the impacts of heterogeneity and tortuosity of actual materials.
- **Implications and Limitations:** This method requires experimental data to validate the model in an experimental process, but this is usually accompanied by certain measurement difficulties, including variations in sample size, boundary effects, and the details of the pore structures. Further, the unstandardization of experimental setups may do this resulting in dissimilarity in the findings

with in-test outcomes, which makes it difficult to compare the models. Although this is the case, experimental validation is still one of the bedrocks in streamlining predictions of ETC models.

A major challenge in developing accurate models for the effective thermal conductivity (ETC) of a porous body is that theory predicts the right behavior. A lot of models depend on assumptions or idealized conditions. For instance, uniform pore structures, constant fluid characteristics, or steady-state conditions. Therefore, it is essential to validate by experiments to evaluate their practical applicability. This section highlights the need for validating models with experimental data, presents the challenges encountered when comparing the two and explains the importance of this validation process in improving and enhancing the ETC models.

Importance of experimental validation

Researching and modeling of effective thermal conductivity of porous media comes with a lot of approximations due to real porous media being quite complicated. Usually, the pore structures in the theoretical model are simplified to ideal structures (like spherical pore, regular packing, etc). Properties of the material phase and fluid phase are uniformly assumed. Nonetheless, actual porous materials are invariably very heterogeneous with respect to pore size distribution, connectivity, fluid saturation, and geometry. As a result of these complexities, there can be considerable differences between the computed and measured ETC.

Thus, experimental validation is crucial for.

- a) Evidence from experiments can be used to check model assumptions like pore shape, material homogeneity, fluid behavior, Zhan et al., (2022). This also validates the effectiveness of the proposed fractal model in prediction of the ETC of porous materials.

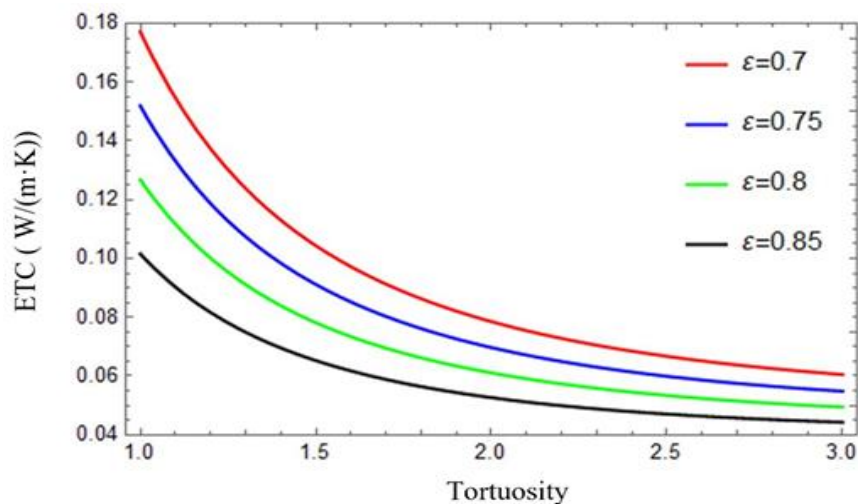


Fig.5. The variation of the ETC with the tortuosity (Zhan et al., 2022).

When tortuosity increases, the effective thermal conductivity decreases. Similarly, an increase in porosity leads to a decrease in the effective thermal conductivity.

- b) Experimental results show areas where models fail to account for real-world factors, such as temperature gradients, anisotropy, and complex pore geometries. Ghanbarian et al. (2015) emphasize that the predictions of these models can be corrected or modified to improve accuracy by comparing them with the measured effective thermal conductivity (ETC) values. This process of model refinement is essential for capturing the complex behaviors observed in porous media.
- c) Experimental data can be used to validate new models. It can serve as an example for subsequent new modelling approaches to ensure that rather than being arbitrary theoretical, those models can end up being physically realized and can reflect any measurement of ETC.

Experimental approaches for validating models

Various testing methods can be used to determine the effective thermal conductivity of porous materials. Different techniques are used for different porous materials and for different conditions under investigation.

- a) Simple experimental techniques can be used to measure a material's important property: its thermal conductivity. Steady-state methods (e.g. guarded hot plate) Dubois and Lebeau (2013) and transient methods [e.g. laser flash (Golombok and Shirvill (1988) or transient line-source techniques (Aichlmayr and Kulacki (2006))] are common techniques in use. These approaches generate data that can be reliably compared with model forecasts, although adjustment may be necessary for porous materials with low conductivity.
- b) To validate models that predict the heat conductivity of engineered porous materials, additively manufactured samples can be used for materials such as metal foams or 3D printed porous structures. (Wang et al., 2021). Materials for which there are no predictive models need experimental data that can be used to validate models.
- c) In case of materials with significant variability in porosity, pore structure and connectivity, such as sintered porous media, experimental measurements provide direct guidance on the ability of the models to predict the ETC under the given conditions (Florez et al., 2013). Results can also assist in evaluating the effect of processing parameters on the sample thermal conductive property like Sintering Temperature.

Discrepancies between theoretical models and experimental data

Sometimes, model predictions fail to match experimental results for a variety of reasons. These discrepancies can be attributed to several factors.

- a) Theoretical models often make simplified assumptions such as uniform pore structures, similar fluid saturation, and negligible effect of boundary. Pore structures are often irregular in reality, and their connections affect routes of heat transfer. Moreover, actual porous media often display thermal conductivity that is not the same in all directions, which is not always captured by isotropic models (Xu et al., 2021).
- b) Many models assume the fluid properties to be constant or overlook temperature-dependent effects. Fluid properties such as viscosity, density, and thermal conductivity will change

with temperature and saturation. This can significantly affect the thermal behavior of porous materials (Shen et al. 2020). There should be a test to prove it under different conditions.

- c) In some cases, the models do not account for the tortuosity of the pore network. Thermally, this tortuosity refers to the complexity of the pathways that heat should travel through. Sintered media has a complex pore network. This can make the effective thermal conductivity lower than what is predicted. Some models do not take tortuosity into account (Florez et al., 2013). Using experimental data, tortuosity effects on heat conduction can be quantified, improving the error margins of models.

Experimental validation of models

Many studies help in validating the models for porous media:

- a) Florez et al. (2013) ran experiments to check the models that sintered porous media are based on. The classical models underestimated effective thermal conductivity for high-porosity sintered porous materials, as compared to experimental results, according to this study. There was a difference in (the thermal) conductivities due to irregularity of pore structures (and) sintering effect. They argued that a better model is needed to take these two factors into account.
- b) Wang et al. (2021) established the effectiveness of their model for additively manufactured metal foams' effective thermal conductivity prediction. Based on experimental data, it was revealed that the Maxwell model performed well for materials with low porosity but required a complex model to predict the high porosity foams' ETC. The study stresses on the fact that porosity and pore connectivity should be taken into account when designing foams as simpler models might not work for foams with high porosity or with non-uniform porosity.
- c) Qin et al. (2019) and Shen et al. (2021) did experiments to validate fractal-based and percolation model for different porous media. Their study showed that the fractal models performed good for materials with irregular pore geometry but there were enormous deviations when applied to materials with simple, regular pore geometry. It is vital to choose a proper model according to the structural features of the material.

HYBRID MODELS AND ADVANCED APPROACHES

This part delves on the current trends in the hybrid models as applied in the past years incorporating components of the theoretical approach, numerical and experimental approach to formulate more precise and robust predictions of the ETC of porous medium. The use of hybrid methods, including a combination of percolation theory with lattice Boltzmann simulations, is a good opportunity to simulate the work of the combined pore geometry and multi-phase fluid interactions.

- **Research Methods and Analysis:** Hybrid models combine the advantages of several methods of modeling to explain numerous variants of factors affecting thermal conductivity including pore size distribution, saturation of fluid, and complexity of geometries. The models seek to fill the gap

between the simplicity in theory and numerically accurate representation by combining several scales of analyzing.

- **Evaluation and Comparison:** The Hybrid models have the ability to surpass the traditional single method models because they offer more predictions that are inclusive. They are however still under development and need further refinement and validation. An example is the incorporation of temperature-dependent fluid property-based hybrid models that have increased the prediction capabilities of ETC under varying conditions including variations in geothermal systems.
- **Implications and Limitations:** All of these mean that, whereas hybrid models provide a more complete model of ETC, they are computational intensive and they may need to base their calibration on large amounts of experimental data. Moreover, these models are characterized by complexity, which further complicates the process of obtaining the results to be interpreted, thus acting as an obstacle to implementing them into the practice.

The effective thermal conductivity of porous media is dominated by temperature and fluid saturation. Many theoretical models either do not take account of these factors or are over simplified in these terms, but experimentally they are important for describing porous materials. Solid matrix and fluid in the pores are characterized by physical properties, which depend of a different temperature -viscosity, thermal conductivity, and specific heat. Likewise, the heat conduction mechanism is affected by the saturation of the fluid in the material. This may be fully saturated, partially saturated or unsaturated. Moreover, the conduction of heat by the solid phase also depends on different physical factors compared to the fluid phase. Temperature and saturation effects to better model ETC of porous media This paragraph addresses the temperature and saturation effects which can improve the modelling of ETC in porous media.

Temperature dependence of effective thermal conductivity

Temperature greatly influences the solid and fluid substances' thermal features in the porous media. In many models, the thermal conductivity of the solid matrix is considered constant, although this is rarely the case, and the thermal conductivity of the solid material varies with temperature. Also, several fluids demonstrate temperature-dependent thermal conductivity. Further, temperature changes can shift energy distribution in phases.

- a) Solid phase conductivity can increase when temperature is raised and this is especially true in metals and ceramics. However, this is more complicated for polymers and other non-metallic materials. In media with high porosity, solid phase may be thin and effect of temperature change on solid conductivity can be high. So, temperature dependence must not be very different from reality to predict ETC properly. Shen and others (2020) highlighted the importance of the temperature dependence of the conductivity of the solid phase and its critical role in modeling as illustrated in equation (11).
- b) In porous media, the conductivity of the fluid phase changes with temperature. As temperature rises, the viscosity of a fluid will generally decrease. The effect on thermal conductivity varies: it will either increase or decrease depending on the type of fluid, which could be, for instance, water, air or oil. Many models overlook these variations, making it

difficult to accurately predict ETCs. For example, in geothermal systems or in situations where the substance is subject to high heat, disregarding the temperature effect on fluid properties can cause substantial divergence from what it should be, Aichlmayr and Kulacki (2005).

- c) In real life, different parts of porous media give off heat in a different manner so there exist temperature gradients in porous media. Assumptions about uniformity of temperature distribution generally don't capture this effect. In heat exchangers or insulation materials, local temperature differences affect thermal conductivity behavior, especially in phase change effects. In other words, materials with changeable or phase-altering composition can display non-linear or state-adjusting thermal conductivity patterns. Failing to account for these effects may cause the models to significantly underestimate or overestimate ETC under non-isothermal conditions, Yang and Vafai (2010).

Fluid saturation effects on effective thermal conductivity

When a porous substance is fully saturated, like with water, it becomes less electrically conductive. When air fills in the gaps, it becomes more electrically conductive again. In fully saturated materials, all pores are filled with fluid, and heat conduction takes place in both solids and fluids. When the saturation level drops, the porous material's heat transfer efficiency takes a hit because the fluid phase stops being continuous and heat must mostly transfer through that material's solid skeleton.

- a) In a porous material having similar amount of solid and fluid phase, both solid and fluid phase will participate in the conduction. The exact contribution can be determined with respect to the thermal properties of the solid and the fluid, as well as their interfacial interaction. Multiple classical models usually assume that the fluid is in thermal equilibrium with the solid matrix and adding their thermal conductivities linearly (e.g., Maxwell's covering). Mere Assumptions Are Not Enough but real-life applications involve thermal resistance at the interface due to interaction between phases. Maxwell was the first to give analytical expressions for effective thermal conductivity of heterogenic medium, as in the following equation:

$$\frac{k_{eff}}{k_m} = 1 + \frac{3\phi}{\left(\frac{k_1+2k_m}{k_1-k_m}\right)-\phi} \quad (16)$$

Where k_m thermal conductivity of the matrix, k_{eff} effective thermal conductivity of the composite.

- b) The Electron Transfer Conductivity (ETC) generally decreases as saturation decreases for partially saturated and unsaturated media. The overall thermal conductivity of the medium decreases with saturation. This is because higher fraction of the conduction occurs through the gas (air or another low-conductivity fluid) which has much lower thermal conductivity as compared to more liquid. When something has no water or little water, air pockets create a thermal resistance for partially filled media, one must consider the impact of meniscus

effects, surface tension, and the non-uniform distribution of the liquid phase. Models based on fractals and percolation theory (Shen et al. 2020) can account for these effects and include considerations for the distribution and connectivity of the liquid phase in the pores.

- c) Many models do not consider the critical saturation threshold for the transition from saturated to partially saturated condition. When saturation levels reach 100%, the material is considered to be a liquid-saturated solid. However, as the liquid fraction decreases, the thermal behavior of the material is dominated by the gas phase. This transition is important in determining ETC and is relevant in unsaturated soil mechanics, drying processes, and some thermal insulation applications (Laimeche et al., 2024). It is important to realize the connection between saturation and ETC in these cases for model improvements.

Models incorporating temperature and saturation effects

More recent models have taken temperature and saturation effects into account when predicting ETC in porous media. These models generally account for how thermal conductivity relates to temperature and saturation, and combines the solid and fluid phases into the overall result.

- a) Fractal models, according to Qin et al. (2019) can take on the irregularity and self-similarity of the pore network, and can take the influence of temperature and saturation into the equation. Such models, for example, can also allow for the fact that pore network structure is a function of temperature or that heat transfer pathways change as fluid saturations vary. Percolation models are also useful for predicting when the behavior may turn go conductive or non-conductive, depending on saturation and temperature.
- b) The hybrid models put forward by Shen et al. (2020), for example, combine classical approaches for calculating effective thermal conductivity with temperature-dependent fluid properties (for numerical simulation). These models utilize the non-linear relations of temperature with fluid properties to enhance the prediction of effective thermal conductivity. Yun et al. (2022) also extended these hybrid models to include a contribution from the surface waves to the heat transfer helping to accurately predict the effective thermal conductivity of porous media.
- c) For working in high-temperature uses, and applications, like geothermal systems, heat exchangers, any models for thermal conductivity must include effects of high thermal gradients, and temperature-induced changes in material properties. These models do not rely on the simplest linear assumption – they use temperature-dependent parameters for model performance prediction, Li et al., (2025).

TEMPERATURE AND FLUID SATURATION EFFECTS ON EFFECTIVE THERMAL CONDUCTIVITY

The effect of temperature and fluid saturation on effective thermal conductivity of porous material shall be discussed in this part. These considerations are important in defining the heat transfer properties of a material in particular in dynamic cases where temperature gradients and saturation of fluids varies with time.

- **Research Methods and Analysis:** The studies using temperature and saturation effects generally include the ETC measurement of materials after placing them at different temperatures and different levels of fluid saturation. Models of these considerations rely on empirical evidence or temperature-related properties of liquids and solids to make greater predictions.
- **Evaluation and Comparison:** Neither of the two above methods can be evaluated using evaluation and comparison: In high-temperature situations, e.g., in geothermal energy systems or heat exchangers, a model with temperature and saturation effects are likely to make more predictions that are accurate. Nonetheless, most models do not consider the nonlinear nature of the thermal conductivity behavior at the extreme conditions and thus there is a mismatch between the predictions and experiment.
- **Implication and limitations:** By introducing the effects of temperature and saturation into ETCs models, the applicability of the ETCs to actual systems is increased. The complexity of modeling these variables however complicates the models to be computationally expensive and hard to apply in a real world. Additionally, the saturation and reduction of temperature effect on the thermal conductivity might not be constant across all the porous materials, and thus the modifications required to be made according to the materials.

The complex shape of many porous materials like irregular pores, fractal-like structures, and heterogeneous compositions often renders traditional models of thermal conductivity inadequate. Simple shapes and sizes of pores lead to the material having a uniform geometry of pore, and hence simple models could work on them. But requirement of thermal conductivity models often does not get fulfilled, as classical models fail to apply as there's an irregularity or similarity involved in the fabric or composition or the geometries. Fractal and statistical models have emerged to address these challenges in the past years. These models can capture the complex geometry of porous media. They will be helpful in predicting the effective thermal conductivity (ETC) of porous materials. The section will examine how fractal and statistical approaches may be applied to model the effective thermal conductivity (ETC) of complex porous media, and their strengths and limitations and applicability.

Fractal models for porous media

Fractal geometry is used by mathematicians to describe objects that are similar at different scales.

The pore structure of many natural and engineered porous materials (like rocks, foams, and sintered materials) display fractal-like properties. Fractal models have got popularity in recent years to represent the complex, multi-scale and irregular pore structures that are typical of these materials Zhan et al., (2022). These models are useful for predicting the ETC of materials with irregular pore networks since they allow for the incorporation of self-similar patterns at different length scales.

Key Features of Fractal Models.

- a) Fractal models believe porous materials have self-similar structures. In other words, the geometry of the material repeats at different scales. This is distinctly true for porous materials with irregular and highly branched pore structures Hu et al., (2020).

Characteristic examples are sintered media and materials with a high degree of porosity. As illustrated in Figure (6) refer to irregular pore structure Hu et al., (2020):

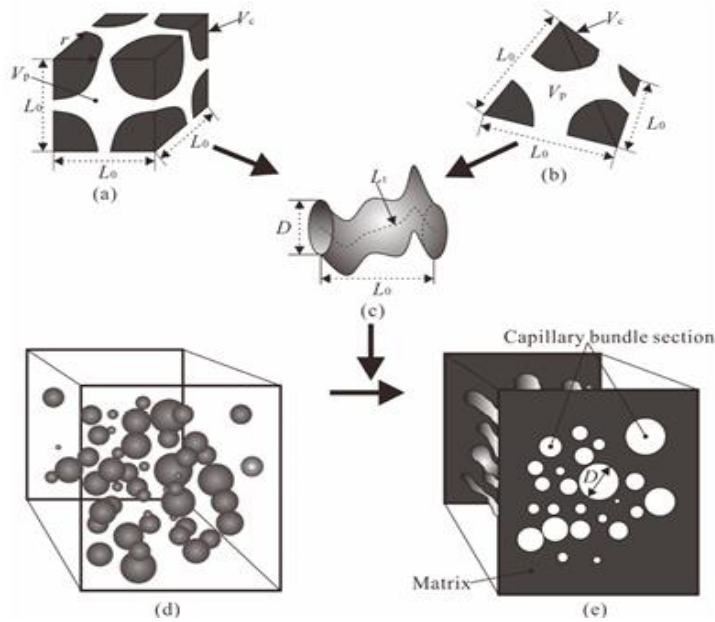


Fig .6. Schematic diagram of irregular pore space transformed into capillary bundle model. (a) conventional loose arrangement; (b) regular compact permutation; (c) capillary channel; (d) actual particle model; (e) transformed capillary model. (a) Conventional loose arrangement; (b) regular (Hu et al., 2020).

- b) The complexity of the pore structure is quantified with the help of fractal dimension. A more complex or convoluted pore structure has a higher fractal dimension while a lower dimension has a more regular geometry. The fractal dimension helps us understand how thermal conductivity of materials changes based on their size and connectivity distributions Zeng et al., (2022). ETC of porous media for fractal model as following in equation (17):

$$k_e = (1 - \phi)k_s + \frac{(1-a^2)^{3/2}(1-\gamma)^2(2-D_f)\phi\lambda_{\max}^{D_T-1}}{(1+D_T-D_f)(1-\phi)L_0^{D_T-1}} \left[k_{nw} - (k_{nw} - k_w) \left(\frac{\lambda_c}{\lambda_{\max}} \right)^{1+D_T-D_f} - k_w \phi^{\frac{1+D_T-D_f}{2-D_f}} \right] \quad (17)$$

Where characteristic length \$L_0\$ is:

$$L_0 = \sqrt{\frac{4A_p}{\pi}} = \sqrt{\frac{D_f(1-\phi)}{\phi(2-D_f)}} \lambda_{\max}. \quad (18)$$

Area of the pore is:

$$A_p = \frac{\pi D_f(1-\phi)}{4\phi(2-D_f)} \lambda_{\max}^2 \quad (19)$$

The Critical Capillary Diameter λ_c , is Determined by:

$$\lambda_c = \left[(1 - \phi^{(3-D_T-D_f)/(2-D_f)}) S_w + \phi^{(3-D_T-D_f)/(2-D_f)} \right]^{\frac{1}{3-D_T-D_f}} \lambda_{\max} \quad (20)$$

And relative roughness γ is:

$$\gamma = \frac{2\phi(h_{\max})\lambda_{\min}^{(2-D)}(1-\alpha^{3-D})}{3\lambda_{\min}^{(3-D)}(1-\phi)} \quad (21)$$

The ETC is a function of porosity ϕ , fractal dimensions for fractal and tortuosity D_f and D_t , relative roughness γ , characteristic length L_o , maximum capillary diameter λ_{\max} , radius amplitude a , and wetting phase saturation S_w . Each parameter in the proposed theoretical model has a distinct physical meaning and can be determined through experiments.

- c) Fractal models are great for considering heat transfer at various scales – Whether examining individual pores or the entire material structure. Fractal models are very effective for materials with irregular pore structures. They accurately model heat transfer at various scales Hu et al., (2020).

Application to thermal conductivity

- Qin et al. established a fractal model to estimate thermal conductivity of a porous medium at various fluid saturation states by Qin et al. (2019). Their model considers the fractal dimension of the pore structure, and the influence of liquid saturation on thermal conductivity. Indeed, when the saturation increases, the thermal conductivity tends towards that of the fully saturated material, but with different evolutions depending on the fractal properties of the pore network.
- Shen et al. (2020) developed ETC model for saturated reciprocal porous media subjected to various temperatures. They demonstrated that this fractal model is able to simulate adequately the ETC of a material with a random pore shape and with a certain level of saturation, when other classic models, such as the Maxwell or the effective medium theory, could not grasp the entire variety of microstructure into the material.

Strengths of fractal models

- Fractal models are very good at accurately representing real-life random structures that make up porous materials. Fractal models offer a better depiction of pore geometry than classical models, particularly for materials with irregular pores, hierarchical structures or heterogeneous distributions, Hu et al., (2020).
- Fractal models can scale their predictions to many different sorts of material like synthetic foam, natural rock, etc. by changing the fractal dimension. These can be used for various applications, including geothermal energy, soil science, and materials engineering, due to their flexibility, Zeng et al., (2022).

Limitations of fractal models

- The prediction accuracy of these models is very dependent on the determination of the fractal dimension. In real-life situations, it can be a challenging task to measure this parameter for heterogeneous or anisotropic materials, especially when they are highly complex. Besides, fractal models generally depend on the hypothesis that an ideal material has perfect self-similarity, which is not always true for real materials, Al-Zainaldin et al., (2016).
- Fractal models of materials usually account for the size distribution of the pores and tortuosity of the material. However, they greatly simplify the connection of pores. In complex porous materials with many inter-connected pores and overall pores, these models, together with pore inter-connection may lead to faulty assumptions, leading to inaccuracies in predictions of ETC, Zeng et al. (2022).

Statistical models for porous media

There is another group, statistical models, to predict ETC of porous materials generally exhibiting random or irregular pore structures. These frameworks use statistical techniques to portray the arrangement of pores and cavities, the combinations of shapes encountered, and proportional significance. Looking at the statistical characteristics of the pore structure, these models can capture the impacts of non-uniformity in porous materials.

Key Features of Statistical Models.

- a) Statistical models often assume that the pore sizes follow a fixed distribution Liu et al., (2021) like normal, log-normal, or exponential. The size of the pores plays an important role in the pathways of thermal conduction and overall effective thermal conductivity Ghanbarian and Daigle (2015).
- b) The porous structure is described as random network of interconnected pores by statistical models which use percolation theory Ghanbarian and Daigle (2015).
These models simulate the thermal conduction in a material with irregular pore shapes.
- c) The ratio of the actual path of a diffusing particle to the length of the straight line connecting the initial and final particle location. Statistical models often use these factors to estimate thermal conduction behavior in pores with an irregular shape, Xu et al., (2021).

Application to thermal conductivity

- Shen et al. (2020) made a statistical model that combined pore size distribution, fluid saturation effects into effective thermal conductivity. The authors predicted ETC in complex heterogeneous pore structures with varied saturation by using statistical methods to model the random distribution of pores.
- Ghanbarian and Daigle (2015) created random pore network system models and analyzed them with percolation theory to model the effective thermal conductivity of porous media. The models allow us to describe and match data from pore networks without needing to know microscopic details, though treating pore geometries realistically will need additional approximation techniques. For partially saturated porous media three conducting phases are

associated with thermal conductivity $\lambda(\theta)$: air, water and solid. Now $\lambda_{dry}(\theta)$ includes contributions from solid and air phases and $\lambda_{sat}(\theta)$, those from solid and water. Thus, air and solid are treated as a single low-conductivity constituent, while water is considered the high-conductivity one. According to P-EMA, the following is proposed, as illustrated in equation (22):

$$(\phi - \theta) \frac{(\lambda_{dry}^{1/t_s}(\phi) - \lambda^{1/t_s}(\theta))}{\lambda_{dry}^{1/t_s}(\phi) + [(\phi - \theta_c)/\theta_c] \lambda^{1/t_s}(\theta)} + \theta \frac{(\lambda_{sat}^{1/t_s}(\phi) - \lambda^{1/t_s}(\theta))}{\lambda_{sat}^{1/t_s}(\phi) + [(\phi - \theta_c)/\theta_c] \lambda^{1/t_s}(\theta)} = 0 \quad (22)$$

$$\theta = \frac{[\lambda^{1/t_s}(\theta) - \lambda_{dry}^{1/t_s}(\phi)] [\theta_c \lambda_{sat}^{1/t_s}(\phi) + (\phi - \theta_c) \lambda^{1/t_s}(\theta)]}{[\lambda_{sat}^{1/t_s}(\phi) - \lambda_{dry}^{1/t_s}(\phi)]} (\phi) \lambda^{1/t_s}(\theta)$$

where $\lambda_{dry}(\theta)$ and $\lambda_{sat}(\theta)$ are the thermal conductivity under fully saturated conditions when the saturating fluid is air and water, respectively, t_s is the scaling exponent, and θ_c is the critical water content at which the high-conductivity component first forms a continuous percolation path.

Strengths of statistical models

- Statistical models of porous media involving random pore structures Kango et al., (2024). Unlike ordinary stats, which conveniently assume a uniform or regular geometry for the pores, the contemporary model can accommodate the random feature of the porous elements.
- statistical models can easily be modified to accommodate a wide range of pore-size distributions and can also account for more sophisticated variations in pore connectivity The flexibility makes them predictive tools for ETC of materials with highly variable or complex pore network Ghanbarian and Daigle (2015).

Limitations of statistical models

- Statistical models may be very effective but they also depend on the precise parameterization of the pore size distribution and the pore network connectivity. It can be difficult especially when the material has very heterogeneous or anisotropic structures Ghanbarian et al., (2013).
- The approximate cost of statistical models relying mostly on Monte Carlo variation and other complex simulations is generally large when applied to big or complex materials. This can limit their practical applicability in some cases Ghanbarian and Daigle (2015).

SIMULATING NUMERICALLY AND COMPUTATIONAL EFFICIENCY IN EFFECTIVE THERMAL CONDUCTIVITY

By using numerical simulations, one can predict the effective thermal conductivity (ETC) of porous media containing complex geometries, heterogeneous structures, and unsteady conditions. Researchers can use methods like lattice Boltzmann (LBM) and finite element analysis (FEA) and other numerical techniques for modeling heat transfer at the pore scale and material scale. These models take into account the complex interactions between the solid matrix and the fluid filling the porous materials. But these techniques are generally computationally intensive, especially for larger size systems or for materials with intricate and irregular pore structures. This section starts by addressing the challenge of computational efficiency in numerical simulations of ETCs, then

discusses strategies to enhance their efficiency. Furthermore, machine learning techniques, reduced-order modeling, and other computational approaches will be explored.

Challenges of numerical simulations for effective thermal conductivity

Simulations that are pore-scale especially numerical ones are highly useful for understanding heat conduction behavior in porous media. However, they face several challenges.

- a) Porous materials have pores that are not very regular taking a few values with respect to size, shape, and orientation commonly referred to as a complex structure. Lattice Boltzmann methods (LBM) and finite element analysis (FEA) can deal with complex geometries. But these methods consume high computational power for large systems with complex pore networks (Yun et al., 2022; Wang et al., 2021). The need for finer grids to capture small-scale features increases the computational cost immensely.
- b) Large-scale simulations modeling the full 3D structure of porous media and solid-fluid phase interaction involve high computation cost especially for the larger 3D systems. When simulating heat transfer over minute or scales, the grids required become extremely fine. Consequently, the number of calculations becomes so large that it may become prohibitive for large systems or for materials with a very fine structure, Wang et al., (2007).
- c) Many simulations need to involve actual applications of porous media, with multiphase (for example, liquid that flows along with gas). For materials with layered or multi-level structures, including different scales (from the instead material scale to the ‘bulk’ material) can be difficult computationally. Running the whole system at high detail takes a very long time and too many resources, Gimenez et al., (2025).

Numerical techniques for effective thermal conductivity

Many numerical methods have been created to model heat transfer in porous materials, each of which has its pros and cons. Two of the most talked about techniques are lattice Boltzmann methods (LBM) and finite element analysis (FEA).

- ❖ **LBM** helps to simulate heat and fluid flow in a wide range of engineering and industrial applications and is becoming an increasingly popular modelling tool. The LBM resolves mesoscopic equations on a discrete lattice grid enabling the modelling of fluid flow and thermal conduction in highly irregular structures. LBM is particularly effective in capturing the local fluid dynamics and temperature distributions within the pores, which is critical for accurate ETC predictions. LBM is being used in works like Yun et al. (2022), whereby it gets combined with percolation theory to accurately estimate heat transfer under porous media with fractal structures.

Strengths

- The Lattice Boltzmann Method gives an accurate and detailed representation of fluid and heat transports in porous media, Liu et al., (2016).
- It is very well helpful in simulating Multi phase flow and heat transfer which is particularly significant in partially saturated porous media, Hassan et al., (2016).
- LBM can manage complicated shapes and boundary conditions, making it perfect for simulating real porous materials, Suga et al., (2009).

Limitations

- Simulations of LBM can cost too much, especially for huge systems or fine-grained simulations.
- Simulations at high resolution (at the micro- or nano-scale) require high computational power and thus, cannot be done with a large size system.

Table 1. Comparison of Different Models for Effective Thermal Conductivity (ETC)

Model	Type	Suitability	Advantages	Disadvantages
Fractal Model (Qin et al., 2019; Shen et al., 2020)	Numerical, Geometrical	Materials with irregular pore structures (such as complex porous or foamed materials)	- High accuracy in modeling irregular pore networks - Effective at different length scales	- Requires precise determination of the fractal dimension, which can be challenging in complex materials- Assumes ideal self-similar materials, which may not be true for real-world materials
Percolation Theory (Xu et al., 2021; Shen et al., 2024)	Numerical	Materials with connected pores (such as fractured rocks)	- Excellent for modeling materials that rely on pore connectivity- Suitable for fractured	- May not perform well for highly irregular or non-spherical pore structures

			materials like rocks	
Maxwell Model(Hsu et al., 1994)	Analytical, Numerical	Materials with regular or low complexity pores (such as foams)	- Suitable for materials with regular pore structures- Easy to use for low porosity materials	- Fails for non-spherical or irregular pores- Not effective for highly complex or fractured materials
Modified Maxwell Model(Behrang et al., 2016)	Analytical, Numerical	Materials like foams or nanoporous structures	- Can be adjusted for irregular or nanoporous materials- Useful for irregular geometries	- May not be accurate for materials with highly complex pore structures
Lattice Boltzmann Method (LBM) (Yun et al., 2022; Suga et al., 2009)	Numerical, Computational	Materials with complex porous structures and multi-phase interactions	- High accuracy in simulating fluid dynamics and heat transfer- Effective for micro and nano-scale simulations	- Computationally expensive- Not feasible for large-scale systems or fine-scale simulations
Effective Medium Theory (EMT) (Gong et al., 2013)	Analytical, Numerical	Materials with simple structures like foams	- Suitable for materials with regular pore structures- Can reflect heat transfer in simple materials	- Not suitable for highly irregular or complex pore structures

The future directions and challenges are itemized as:

- 1. Incorporating Realistic Pore Geometries:** Future models need to consider realistic pore complex geometries instead of simplified uniform models. Detailed geometries obtained from advanced imaging techniques, for example, 3D imaging /X-ray tomography can be used to improve predictions for complex materials.
- 2. Advanced computational methodologies:** The accurate prediction of thermal conductivity models can be enhanced by applying machine learning and deep learning methods. These models can accurately represent complex relationships between material

properties, pore structures, and the thermal behavior, and can be calibrated based on large experimental datasets.

3. Challenges in Model Validation:

- **Measurement Sensitivity:** Accurate measurements of ETC in porous materials are difficult due to variability caused by sample size, boundary effects, and inconsistent experimental conditions. This creates challenges in comparing models with real-world data.
- **Scale Effects:** Discrepancies may arise due to the scale difference between micro- or mesoscale models and macro-scale experimental measurements, where different heat conduction mechanisms may dominate.
- **Complex Real-World Applications:** Real-world applications often involve dynamic conditions such as temperature gradients, variable fluid flow, and multi-phase interactions, making it difficult to model their thermal behavior accurately.

4. Complex Fluid-Structure Interactions: Models are required to represent interactions between a fluid and solid phase that are non-equilibrium, such as phase changes or thermal expansion/contraction of the material that are frequently neglected in classical models.

5. Limited data for complex models: Numerous models are already available and need extensive experimental data for calibration and validation of temperature-dependent parameters. Further research under different operation conditions is required to make these models more robust.

6. Multiphases and Multi-scales Modeling: Next future models have to include the multiphase fluid distribution (liquid-gas, liquid-solid) and the multi-scale effects, describing the effect of the micro-scale fluid behavior for the macro-scale thermal conductivity.

CONCLUSIONS

The effective thermal conductivity (ETC) of porous materials is a parameter of great importance that determines the functionality of materials, in a very extensive range of engineering tasks, such as thermal insulation, heat exchange systems, and geothermal energy systems. Proper modeling of ETC of porous materials is critical in the optimization of thermal control in such applications, but it is a complicated and heterogeneous process because of the presence of porous media. This review was supposed to discuss the constraints of the current thermal conductivity models, as well as to provide an explanation of the importance of the application of proper models to different kinds of porous materials, depending on their peculiar features like geometry of the pore, fluid saturation and heterogeneity of the material.

We have reviewed a variety of ETC prediction methods in our review, such as theoretical models, numerical simulations, experimental methods, and hybrid models that are combinations of several methods. We have established that, as simple and easy to apply as they may be was, theoretical models (e.g. the Maxwell model and the Zehner-Schlunder model) are not very good at modeling complex porous structures especially when their geometry is irregular. Lattice Boltzmann simulation and other numerical methods like the finite element analysis are more accurate, but computationally intensive, and thus cannot be used in many cases. Experimental validation is still needed to improve their models because, under different conditions, it will allow acquiring the required data on the actual behavior of porous materials in different conditions. In addition, the combination of methodologies has also proven expensive in the boundaries of single approaches, and hybrid models also need to be improved and validated.

Based on the implications of our results, the investigation of stronger models that are able to embrace the entire spectrum of a range of physical phenomenon seen in porous materials such as dependence on temperature, effects of fluid saturation, and intricate pore architectures should be investigated. It is also becoming urgent to have models that can predict ETC of a wider set of available materials, including nanoporous and fractured media, and which are increasingly significant in the field of modern engineering.

In the future, the optimistic opportunities of these tools seem to be the incorporation of advanced computational methods, including machine learning and deep learning, to improve the forecasting ability of ETC models. These methods can be used to maximize the parameters of the model and enhance accuracy using large experimental datasets. In addition, availability of superior imaging technology that is currently rising, like X-ray tomography, provides new avenues of data collection of realistic pore geometry that may greatly enhance prediction of models.

Finally, future studies of the area, should aim at combating the difficulty that heterogeneous and complex in the real world porous materials entail. As the knowledge of how solid and fluid phases interact with porous media further develops, coming up with more effective, accurate, and universally applicable models of predicting effective thermal conductivity would be important in the future to further the design and effectiveness of a wide range of materials in various applications.

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