# STABILITY AND DYNAMIC ANALYSES OF PIPE CONVEYED FLUID STIFFENED BY LINEAR STIFFNESS ELEMENT USING FINITE ELEMENTS METHOD

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## ABSTRACT

Different types of dynamic instability of the system occur when the flow velocity exceeds a specific value; thereby the problem of dynamic instability of the pipe conveyed flow became shining. Several effective parameters play an important role in stabilizing the system, such as stiffness addition. In this search, dynamic analysis of pipe conveying fluid stiffened by linear spring was studied using the finite element method. The effect of stiffness addition (linear spring) and the effect of spring location was deeply studied. Also, the effect of flow velocity on the dynamic stability of the system was taken into the consideration. It was observed that there is a critical flow velocity after which the system loses its stability. There is a critical spring constant at which the dynamic behavior becomes more sensitive. Also, there is a specific spring location (effective location) so the spring offers the best results for the frequency of the system. Effective spring locations depend on the flow velocity and spring constant itself. The effective spring location was found to be arranged from *x/l*= 0.7 to 1.

**Keywords: Stability, Dynamic analysis, Finite element; Pipe conveyed fluid; Spring addition; Stiffness ratio**

## NOMENCLATURE

| Latin Symbol | Description | Units |
|---|---|---|
| A | Cross-sectional flow area | $m^2$ |
| e | Element | - |
| E | Modulus of elasticity of pipe | $N/m^2$ |
| F | Reaction force inside the pipe | N |
| g | Acceleration constant | $m/s^2$ |
| I | Pipe second moment of area | $m^4$ |
| [I] | Unity matrix | - |
| $[\hat{k}_1]$ | Stiffness matrix of pipe | - |
| $[\hat{k}_2]$ | Stiffness matrix comes from flow around deflected pipe | - |
| l | Length of the pipe | m |
| M | Fluid mass per unit length | kg/m |
| m | Pipe mass per unit length | kg/m |
| $\breve{M}$ | Bending moment | N/m |
| $N_i$ | Shape function | m/s |
| p | Pressure inside the pipe | m/s |
| Q | Shear force | N |
| $\breve{q}$ | Wall shear stress | $N/m^2$ |
| q | Lateral displacement of the pipe | m |
| $\dot{q}$ | Lateral velocity of the pipe | m/s |
| $\ddot{q}$ | Lateral acceleration of the pipe | $m/s^2$ |

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| Latin Symbol | Description | Units |
|---|---|---|
| *S* | Pipe inner perimeter | m |
| T | Tension force in the pipe | N |
| t | Time | S |
| U | Fluid velocity relative to the pipe | m/s |
| x,W | Cartesian axes | - |

## INTRODUCTION

The subject of flow-induced vibration was started when the industry was transformed from a primary and simple to a more complex one. The first observation of flow-induced vibration was started from our houses and then keeps going to the nuclear stations. All of us notice how the water pipe moves (vibrates) in the garden when the water flows through it but we didn't take it into account because its motion was ineffective and under control. The problem of flow-induced vibration seemed to be more dangerous and maybe uncontrolled in the case of transporting lines in the nuclear stations. With regard to the foregoing, the different studies on flow-induced vibration were presented. One of the most important studies on the structure especially those supplied to dynamic external or internal applied loads is the dynamic behavior. So great and deep studies on the dynamic behaviors of the structure were accomplished. It was so important to explain all the effective parameters and understand how they affect the dynamic behavior itself. In simple structures, one can predict some results of the dynamic response due to the accumulated experience or sometimes what is related to the common experiments that occur during our lives. When the structure becomes more complex such as a pipe conveying fluid, the dynamic behavior becomes more difficult to predict. The complexities are arising for the combination of the different parameters resulting from pipe stiffness (structural stiffness) and flow stiffness (hydrodynamic stiffness resulting from the velocity of flow). The combined effect of these stiffnesses represents the major source of complexity. Flow-induced vibration takes great attention of the researchers. Interest in studying the dynamic behavior of fluid conveying pipes was stimulated when excessive vibrations were observed and subsequently analyzed first by Ashley and Haviland, 1950. Long, 1955, studied the influence of clamped-clamped and clamped-pinned boundary conditions on the critical velocity. Gregory and Paidoussis, 1966 presented results on the dynamic behavior of a cantilevered pipe conveying fluid. Chen, 1971, considered the critical flow velocity of a cantilevered pipe with a spring attached at its free end. Sundararajan, 1974 and Sugiyama *et al*, 1975discussed the effect of spring support at the free end of elastic systems subjected to a follower force. Neimark, 1978 and Neimark *et al* 2003 used the D-composition method in the analytical solution of the pipe conveyed flow problem, and they mentioned that the main disadvantage of the method is the necessity to know the number of unstable modes. In addition, they added that in this particular case, a direct numerical calculation is not stable since it would give a big error through calculating the imaginary part. Becker, *et al* 1979, examined the dynamics of the system of end spring support. Sugiyama, et al, 1985, studied the dynamic behavior of the pipe conveyed fluid with spring support. They stated that the support location has a great effect on the dynamic characteristics of the structure. Païdoussis, 1998, claimed that "An extension to studying the dynamic behavior with spring support take at the free ends (no intermediate support location) and anyone didn't mention why?. The main reason was the complexities in the theoretical modeling or inaccuracy of the results". Also, he mentioned that "The case when there is an additional, intermediate spring support is a very complex problem, and it is an arduous task". Paidoussis, 2008 studied the dynamic behavior of fluid conveying pipe. Jixing Yang, *et al*, 2009, studied the dynamic analysis of fluid-structure interaction on cantilever structures using the Ansys package to obtain the dynamic characteristics. Lu, and Lee, 2009 studied the dynamic instability of the of pipe conveying fluid. They showed that the results gave good agreement with the analytical method. Stephanie, *et al*, 2010, accomplished the theoretical analysis of microscale resonators containing internal flow. Huang Yi-min, *et al* 2010, studied the natural frequency analysis of fluid conveying pipelines with different boundary conditions using eliminated element-Galerkin method. They explained that the Galerkin method gives good results with ineffective errors. Ni, *et al*, 2011, used Differential Transformation Method (DTM) to analyze the free vibration problem of pipe conveying fluid. They demonstrated that the DTM method gave good precision as compared with the analytical methods. In light of the aforementioned literature about the flow-induced vibration thorough a pipe conveying fluid, one may reformulate and explain the main points of interest as follows: -

1- The problem of flow-induced vibration (pipe conveyed fluid) plays a very important role in the engineering fields, such as nuclear plants, aviation, cosmonautics, oil transportation, municipal water supply, etc.
2- The problem was solved analytically by different methods such as the Decomposition method by Neimark 1978, DTM by Ni, et al. 2011by, and direct Euler method by Sugiyama, et al 1985 as an example. They referred that there were some disadvantages to using the analytical method. As an example, Sugiyama, et al 1985mentioned in their paper that "The spring at the tip end provides a serious case where the eigenfunction in the analytical solution does not satisfy all the boundary conditions”. Also, "The accuracy of the results are different with the difference of the spring value using the analytical methods of the solution".

Therefore, the aims of the presented paper are to use FEM to eliminate the error arising from using the analytical method discussed previously and to study the effect of spring addition and its location on the dynamic behavior of pipe conveying fluid secondly.

## DERIVATION OF GOVERNING DIFFERENTIAL EQUATION

The straight pipe is very important in engineering applications, especially that of cantilever constraint. The main reason of study the cantilever system is that the cantilever pipe conveying fluid is one of the ways of realizing a follower force situation and thus, the pipe is not the conservative system**,** Sugiyama, *et al*, 1985. Then, a cantilever straight pipe will be adopted in this study. Figure(1) presents a cantilever pipe conveyed fluid with a fluid velocity of *U* m/s and length *l*. *x/l* represents the distance from the spring to the cantilever constrained. Pipe of (2 m) length. The fluid and pipe densities are 1000 kg/m$^3$ and8000 kg/m$^3$, respectively. The pipe thickness is assumed to be 0.001 m with outer diameter of 0.01 m. The elastic modulus of pipe is 207 GPa. Païdoussis 1998, Païdoussis, *et al* 2011, and Mustafa, 2011, list the step-by-step procedure to derive the governing differential equation of motion of the straight flexible pipe conveying fluid. The system consists of a uniform pipe of length (*L*), pipe mass per unit length (*m*), flexural rigidity (*EI*), conveying fluid of mass per unit length (*M*), flowing axially with fluid velocity (*U*). The cross-sectional flow area is (*A*), the inner perimeter is (*S*) and the fluid pressure is (*p*). From Figure (2), three partial differential equations parallel to the directions of the elements are resulted due to applying the equilibrium equations in the corresponding directions. It is so easy to carry out the three equilibrium equations and its mentioned in detail in Païdoussis (1998, 2011). It is not important to mention them here, but the important is the overall equilibrium equation which resulted from combining the three equations as shown in Eq. (1);

$$EI\frac{\partial^4 w}{\partial x^4}+MU^2\frac{\partial^2 w}{\partial x^2}+2MU\frac{\partial^2 w}{\partial x\partial t}+(m+M)\frac{\partial^2 w}{\partial t^2}=0 \tag{1}$$

$\sum_{i=1}^{4} N_i(x)q_i$

To fix the idea and to better understand the equation deeply, the terms in Eq.(1) will be defined. The term ($EI\frac{\partial^4 W}{\partial x^4}$) represents a force component acting on the pipe as a result of pipe bending. The expression ($MU^2\frac{\partial^2 W}{\partial x^2}$ ) represents the force component acting on the pipe as a result of flow around a deflected pipe (curvature in the pipe). Some researchers refer to this term as a centrifugal force of the fluid element due to its instantaneous velocity and instantaneous curvature of the deflected pipe. This, the term has a great effect on the pipe stability and accelerates the pipe to be unstable. The term ($2MU\frac{\partial^2 W}{\partial x \partial t}$) represents the inertial force associated with the Coriolis acceleration arising from the fluid flows with velocity $U$ relative to the pipe. The inertia force for both fluid and pipe density is referred in term ($(m+M)\frac{\partial^2 W}{\partial t^2}$). As a final sight, it was noted that the dynamic behavior of the system largely depends on the elastic stiffness of the pipe, flow velocity, and lateral displacement (which mean the boundary conditions). So, any change in the system stiffness should change the dynamic behavior. Thereby, the addition of spring (stiffness) is of majority importance. The boundary conditions for the cantilever pipe are;

$$W\Big|_{x=0}=0 \qquad \frac{\partial W}{\partial x}\Big|_{x=l}=0\,, \qquad EI\frac{\partial^2 W}{\partial x^2}\Big|_{x=l}=0\,, \qquad EI\frac{\partial^3 W}{\partial x^3}\Big|_{x=l}=0\,,$$

## FINITE ELEMENT FORMULATION

From the literature and the highlight of the previous section, it was noted that there were some difficulties in the analytical methods for studying dynamic behavior. Due to the complexity in solving the differential equation of pipe conveyed fluid [Eq (1)] (higher partial and higher-order differential equation), then FEM will adopt to obtain the dynamic behavior as a numerical technique using quadrilateral element. The displacement field in element direction can be therefore written as follows (Raw, 2004 and Mustafa, 2011);

$$W(x) = \sum_{i=1}^{4} N_i(x) q_i \tag{2}$$

where

$q_i$ the generalized coordinates.

$N_i$ the shape functions which can be evaluated as:

$$N_1 = \frac{1}{l^3}(2x^3 - 3lx^2 + l^3)$$

$$N_2 = \frac{1}{l^2}(x^3 - 2lx^2 + l^2 x)$$

$$N_3 = \frac{1}{l^3}(3lx^2 - 2x^3)$$

$$N_4 = \frac{1}{l^2}(x^3 - lx^2) \tag{3}$$

Where, $l$ is the element length. The kinetic and potential energies of the pipe element can be expressed by :

$$T = \frac{1}{2}\int_0^l (M+m)\left(\frac{\partial W}{\partial t}\right)^2 dx = \frac{1}{2}\sum_e \dot{q}^T (M+m) \int_0^l N^T N \, dx \, \dot{q} \tag{4}$$

$$V_1 = \frac{1}{2}\int_0^l EI\left(\frac{\partial^2 W}{\partial x^2}\right)^2 dx = \frac{1}{2}\sum_e q^T EI \int_0^l \bar{\bar{N}}^T \bar{\bar{N}}\, dx\, q \quad . \tag{5}$$

Each prime sign that appears above the shape function symbol, i.e. "*N*", represents a one-time derivative with respect to x-coordinate. Thus, mass ($\widehat{m}$) and stiffness ($\hat{k}_1$) matrices are equal to (Rao, 2004)

$$[\widehat{m}] = \frac{(m+M)\,l}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & -4l^2 \end{bmatrix} \tag{6}$$

$$[\hat{k}_1] = \frac{2EI}{l^3}\begin{bmatrix} 6 & 3l & -6 & 3l \\ 3l & 2l^2 & -3l & l^2 \\ -6 & -3l & 6 & -3l \\ 3l & l^2 & -3l & 2l^2 \end{bmatrix} \tag{7}$$

The term ( $MU^2\frac{\partial^2 W}{\partial x^2}$) has potential energy that can be represented in terms of displacement shape function derived for the pipe as;

$$V_2 = \frac{1}{2}\int_0^l M\, U^2 \left(\frac{\partial W}{\partial x}\right)\left(\frac{\partial W}{\partial x}\right) dx = \frac{1}{2}\sum_e q^T MU^2 \int_0^l \bar{N}^T \bar{N}\; dx\; q \tag{8}$$

The stiffness matrix that comes from flow around the deflected pipe is

$$[\hat{k}_2] = \frac{MU^2}{30l}\begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & -l^2 \\ -36 & -3l & 36 & -3l \\ 3l & -l^2 & -3l & 4l^2 \end{bmatrix} \tag{9}$$

It is important to clarify that the stiffness matrix $[\hat{k}_2]$ leads to weakening the overall stiffness of the pipe system (Païdoussis, 1998). While the expression ($2MU\frac{\partial^2 W}{\partial x \partial t}$) represent the Coriolis force which causes the fluid in the pipe to whip can be represented by dissipation energy as;

$$\mathcal{R} = \frac{1}{2}\int_0^l 2MU \left(\frac{\partial W}{\partial x}\right)\left(\frac{\partial W}{\partial t}\right) dx = \frac{1}{2}\sum_e q^T\; 2MU \int_0^l \bar{N}^T N\, dx\, \dot{q} \tag{10}$$

This gives the unsymmetrical damping matrix

$$[\hat{C}] = \frac{MU}{30}\begin{bmatrix} -30 & -6l & -30 & 6l \\ 6l & 0 & -6l & l^2 \\ 30 & 6l & 30 & -6l \\ -6l & -l^2 & 6l & 0 \end{bmatrix} \tag{11}$$

## DYNAMIC ANALYSIS

The standard equation of motion in the finite element form is

$$[m+M]\{\ddot{q}\} + [\hat{C}]\{\dot{q}\} + [k_{total}]\{q\} = \{0\} \tag{12}$$

Where $k_{total} = \hat{k}_1 - \hat{k}_2$.Since the above equation has a damping term with skew-symmetric characteristic, thus the solution of the eigenvalues problem should be executed to the characteristic matrix [ $\Omega$ ] (Meirovitch, 1980), which is equal to

$$[\Omega]=\begin{bmatrix}[0] & [I]\\ -[m+M]^{-1}[k_{total}] & -[m+M]^{-1}[C]\end{bmatrix} \tag{13}$$

The solution of eigenvalue problem yields complex roots. The imaginary part of these roots represents the natural frequencies of the damped system. The real part indicates the rate of decay of the free vibration.

## RESULTS AND DISCUSSION

Two important steps must be taken into the consideration before extracting the results and discussion. The first is the convergence criterion for selecting a suitable number of elements at which the results converge. The second is the verification for the accuracy of the results and agreement with other researchers.

### Convergence criterion

The number of elements represents the major parameter for the accuracy of the results and the consumed time to solve the problem. Some types of errors such round off error is increase or decrease with increasing or decreasing elements number. Therefore, it was observed that there is a critical elements number at which the result becomes converges, Waheed, 2006. Table (1) presents the best numbers of elements. It is noted that ten elements gave a convergence in the results and then they will be used to discretize the pipe system.

### Verification and agreement of the results

Table (2) presents the comparison with another researcher. From the table, it was noted that there are good agreements between the results as compared with Jixing Yang, et al, 2009 in his study about the dynamic Analysis of fluid-structure interaction on cantilever structure without adding a spring.

### Effect of spring and its location on the frequency

Figures (3-6) present the effect of spring location and dimensionless spring constants ($K=kl^3/EI$) on the natural frequency of cantilevered pipe conveying liquid. The figures are remarked as data points for easier comparison. As a general view, it was noted that the frequency is increased with increasing the dimensionless spring constant. For the same value of spring, the frequency is seemed to be increased with an increased $x/l$ ratio (spring location measured from the constraint). This behavior was dominated for a specific spring constant started from $K$=5 to 28 so the maximum frequency occurred at the free end of the cantilever and at $x/l$=1. With increasing the dimensionless spring constant $K$=32 as an example, a different behavior was observed. Where an inflection point, at which the frequency has a maximum value then after it will decrease, is brought to light. One can observe that the critical spring location ($x/l$) moved from the free end and directed to the constrained end with increasing spring constant above its critical value. For the aforementioned, two interesting features were observed. First; there was a critical stiffness value in which the frequency response is started to change. The second; a critical spring location at which the frequency attains its maximum value is presented. From the other side, the bandwidth of the frequency (the difference between two sequenced values) seemed to be increased gradually with increased $x/l$ until the critical spring location and decreased after that location. This means that the effectiveness of the spring increases towards the critical location and decreases far from it. Also, it was remarked that from the results, the bandwidth of the frequency values (between two sequenced values and same $x/l$ value) was increased with increased spring value and reached its maximum value at the critical spring location. In the other hand, this difference was reduced with increasing the spring value; as shown in Figure (3), Figure (4), Figure (5), and Figure (6), respectively. So, in Figure (6) the differences can be neglected for larger spring values. Figures (4, 5, and 6) present the relationship of the frequency and spring location for different spring values at 5, 8, and 13 m/s of fluid flow, respectively. The general sight is the frequency was decreased with an increase in the flow velocity. The frequency of the system largely depends on the structural stiffness of the pipe. With increasing the flow velocity, the hydrodynamic stiffness resulting from increasing the flow velocity (as mentioned in Equation (9) is increased also due to the direct relationship between them. The hydrodynamic stiffness leads to weakening the structural stiffness of the pipe. Then increasing the flow velocity means increasing hydrodynamic stiffness and thereby decreasing the pipe stiffness which leads to a decrease in the frequency. An alternative way of the effect of flow velocity on the frequency can be shown in Figure (7). The more distinctive feature for Figures (3, 4, and 5) is that all spring constants have an effect on the frequency at all values of *x/l* while in Figure (6) there were specific spring constants that have no effect on the dynamic behavior for a specific pipe length. As an example, for a spring constant of *K*=5, the frequency value is equal to zero for x/*l* = (0 to 0.8) as shown in Figure(6a). Also, similar cases can be noticed from Figure (6). The domain *x/l* between (0 and 0.8) can be called ineffective spring location which means that the addition of *K*=5 at a location arranged from (0 to 0.8) percent of the pipe length has no effect on the dynamic characteristics of the system. Then, the rest of the pipe length can be called effective spring location. The ineffective and effective spring location largely depends on the spring constant and fluid velocity. One can observe ineffective spring location increases when the spring constant is small and decrease with larger values. The presence of ineffective spring location is a strong guide on the coupling of flow (fluid velocity) and structure (spring addition). A comprehensive study about effective and ineffective spring locations was accomplished and listed in Table (3) in the next section.

**Effective and ineffective spring location**

Table(3) shows effective spring location for different values of flow velocity and dimensionless spring constants. The symbol (-) means no effective spring location so no change in the dynamic characteristics with and without spring addition (addition of spring at any location of the pipe is meaningless). From the first sight, effective spring location has direct proportion with spring constant and inverse with the flow velocity. Another point of interest, for a specific flow velocity such as 24 m/s, the spring constant of K=60 has no effect on the dynamic response of the pipe from *x/l*= 0 to *x/l*= 0.96 so the effective spring location is 0.04 of the pipe length from the free end is the effective spring location. The more surprising result from the table was at 25 m/s of flow velocity, there is no effective spring location (-) for all spring values. This means at a specific flow velocity, any addition of spring whatever its value small or high leads to the same results so there is no change in the dynamic characteristics.

## CONCLUSIONS

The following are some conclusions that can be drawn from the obtained results

1. Spring addition plays an important role in unchanging the dynamic behavior of the cantilevered pipe conveying fluid.
2. Spring location has a great effect on the results of the system stability.
3. There is a critical spring value at which the frequency attains its maximum value.
4. There is an effective and ineffective spring location and is largely depends of the spring constant and flow velocity.
5. At a specific flow velocity, any addition of spring whatever its value small or high leads to the same results so there is no change in the dynamic characteristics.

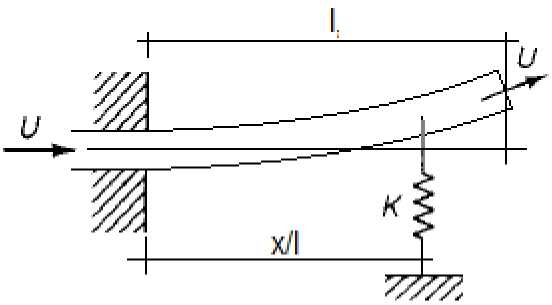



Fig1. Cantilever pipe with intermediate spring support *K*.

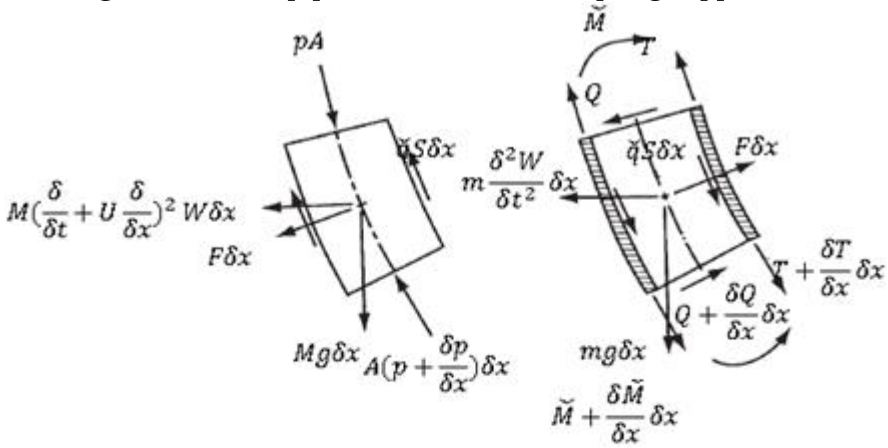


Fig 2. Presents fluid and pipe elements and reaction forces and moments (Mustafa, 2011)

Table 1. presents the best numbers of elements.

| Element numbers | Fundamental frequency (Hz) |
|---|---|
| 2 | 4.12 |
| 4 | 4.099 |
| 6 | 4.097 |
| 8 | 4.095 |
| 10 | 4.094 |
| 12 | 4.094 |

Table 2. presents the comparison of results.

| | Dimensionless Natural frequency at different geometrical ratios | |
|---|---|---|
| r/h | Present work | Jixing Yang, et al, 2009 |
| 1/5 | 7.2471 | 7.2241 |
| 1/10 | 2.4155 | 2.4034 |
| 1/20 | 0.6279 | 0.6229 |

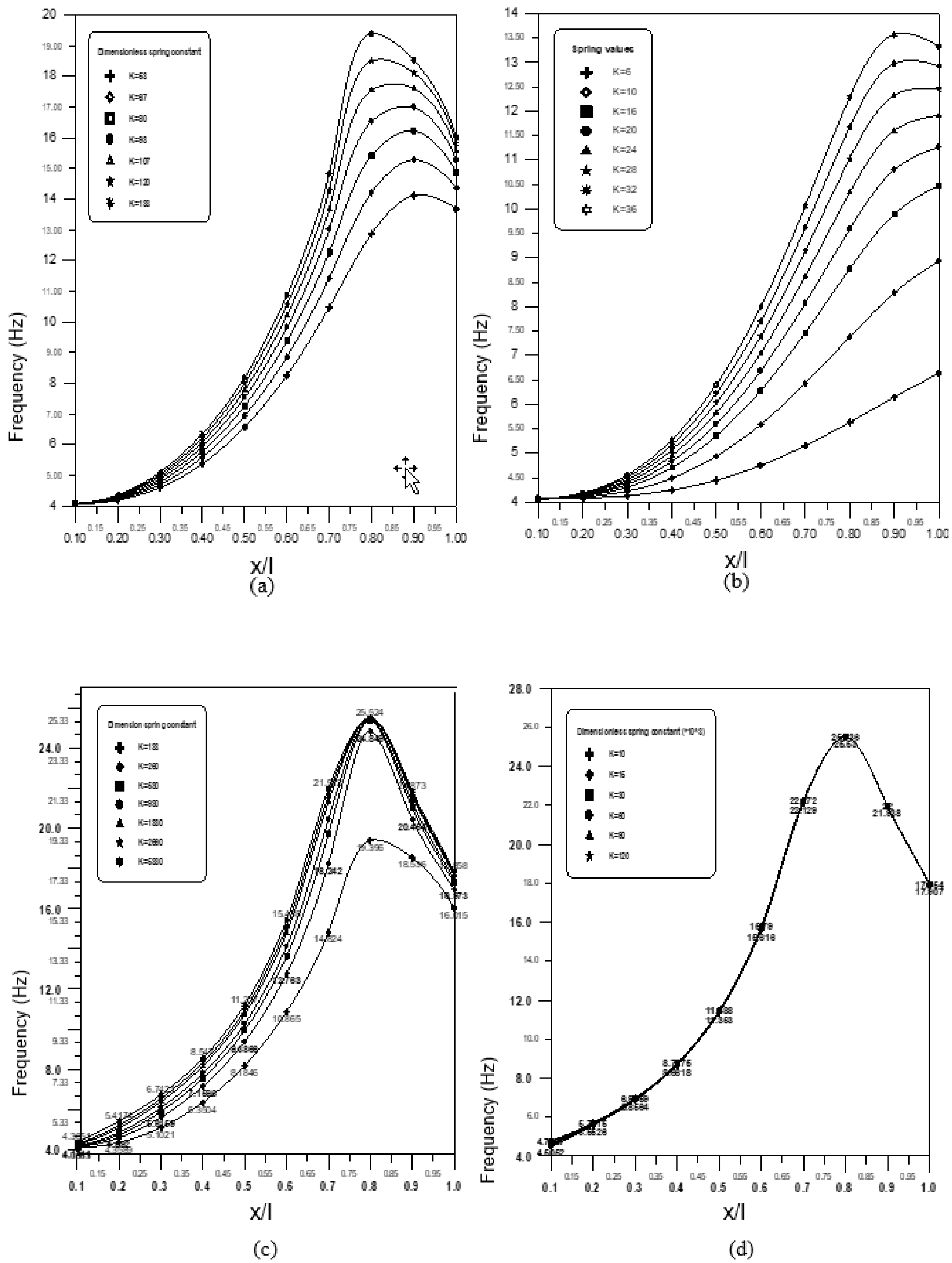



Fig.3. Presents effect of dimensionless spring constant and spring location on the system frequency with no fluid velocity (*U*=0).

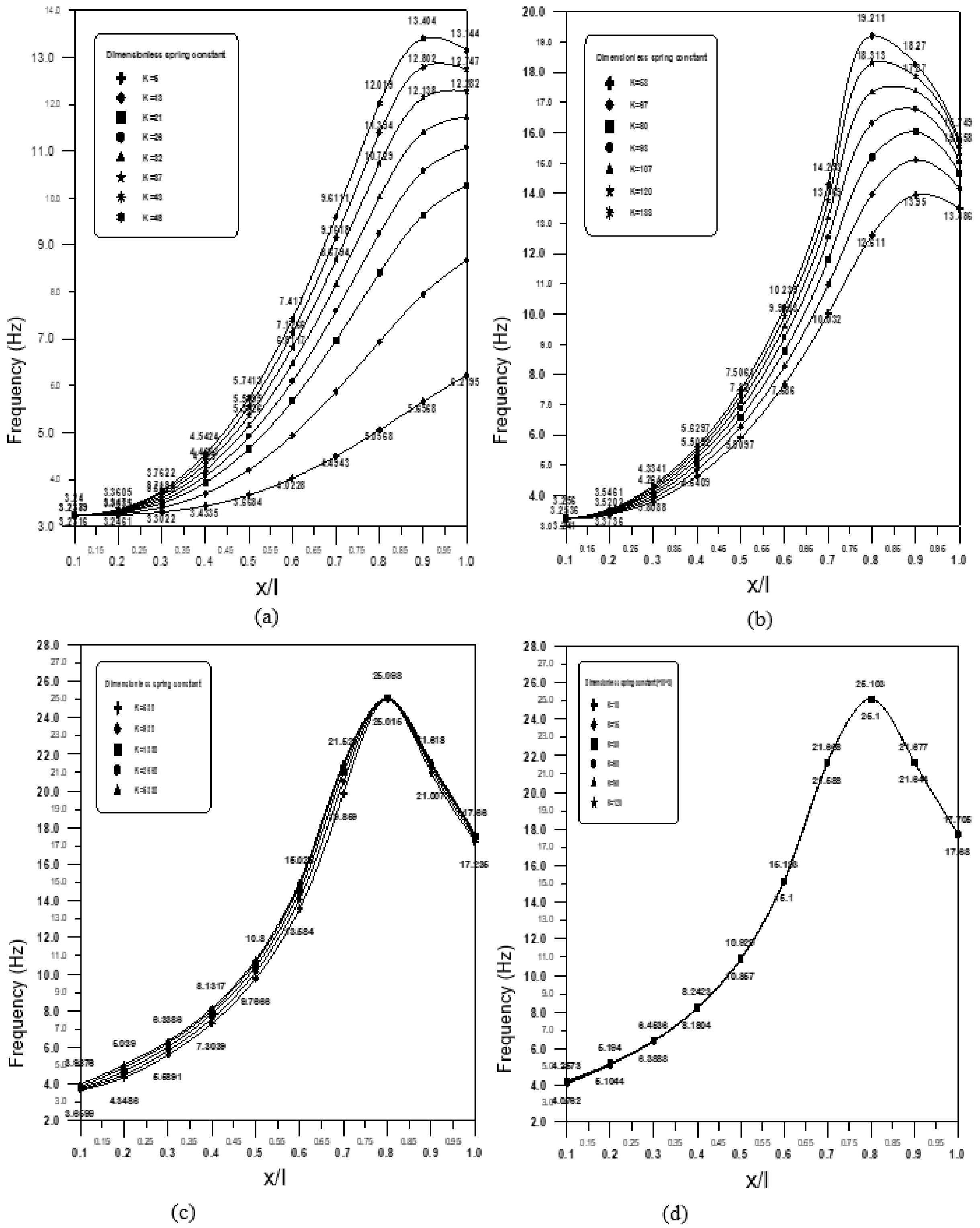



Fig.4. Presents effect of dimensionless spring constant and spring location on the system frequency with fluid velocity $U$=5 m/s.

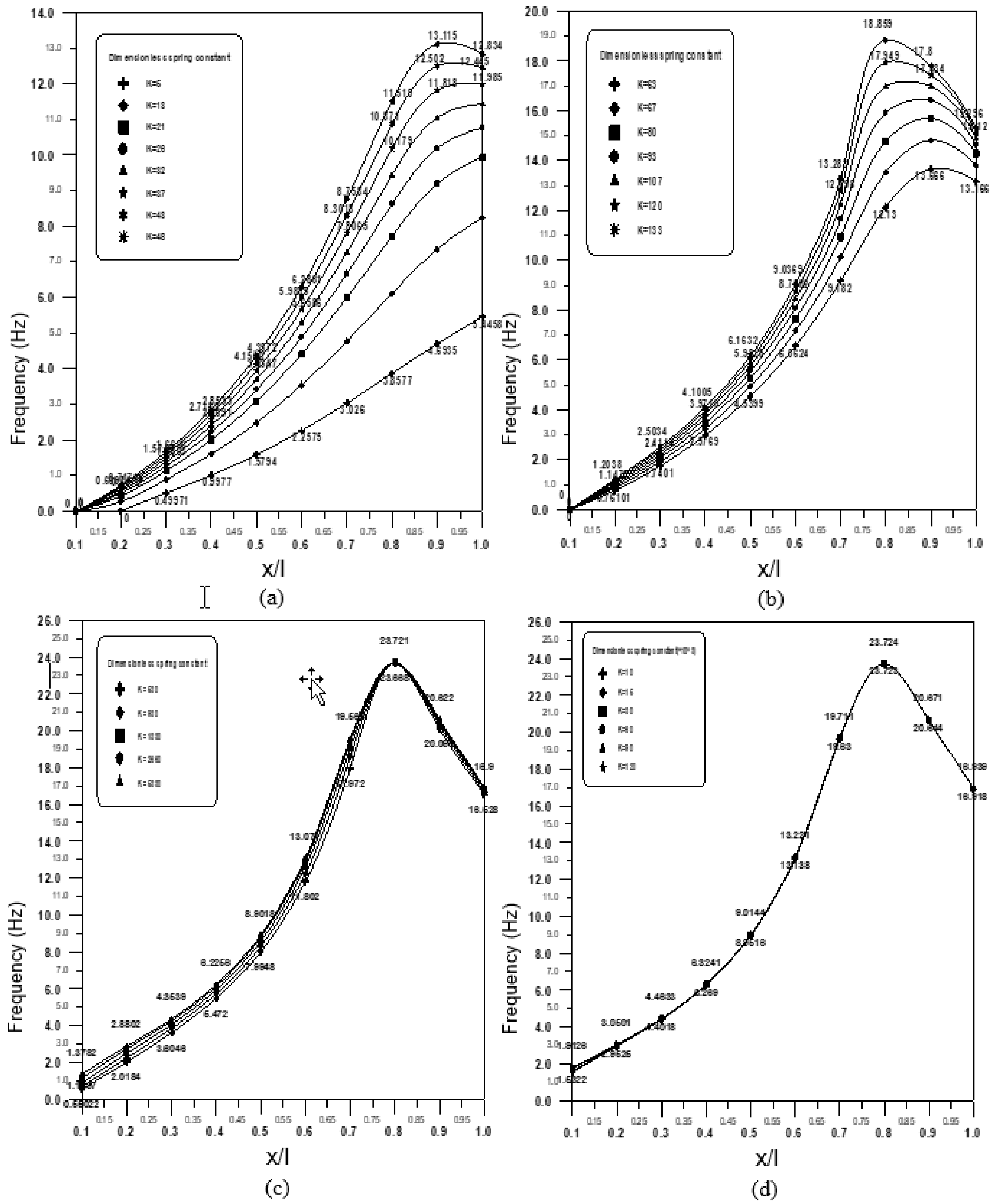



Fig.5. Presents effect of dimensionless spring constant and spring location on the system frequency with fluid velocity $U$=8 m/s.

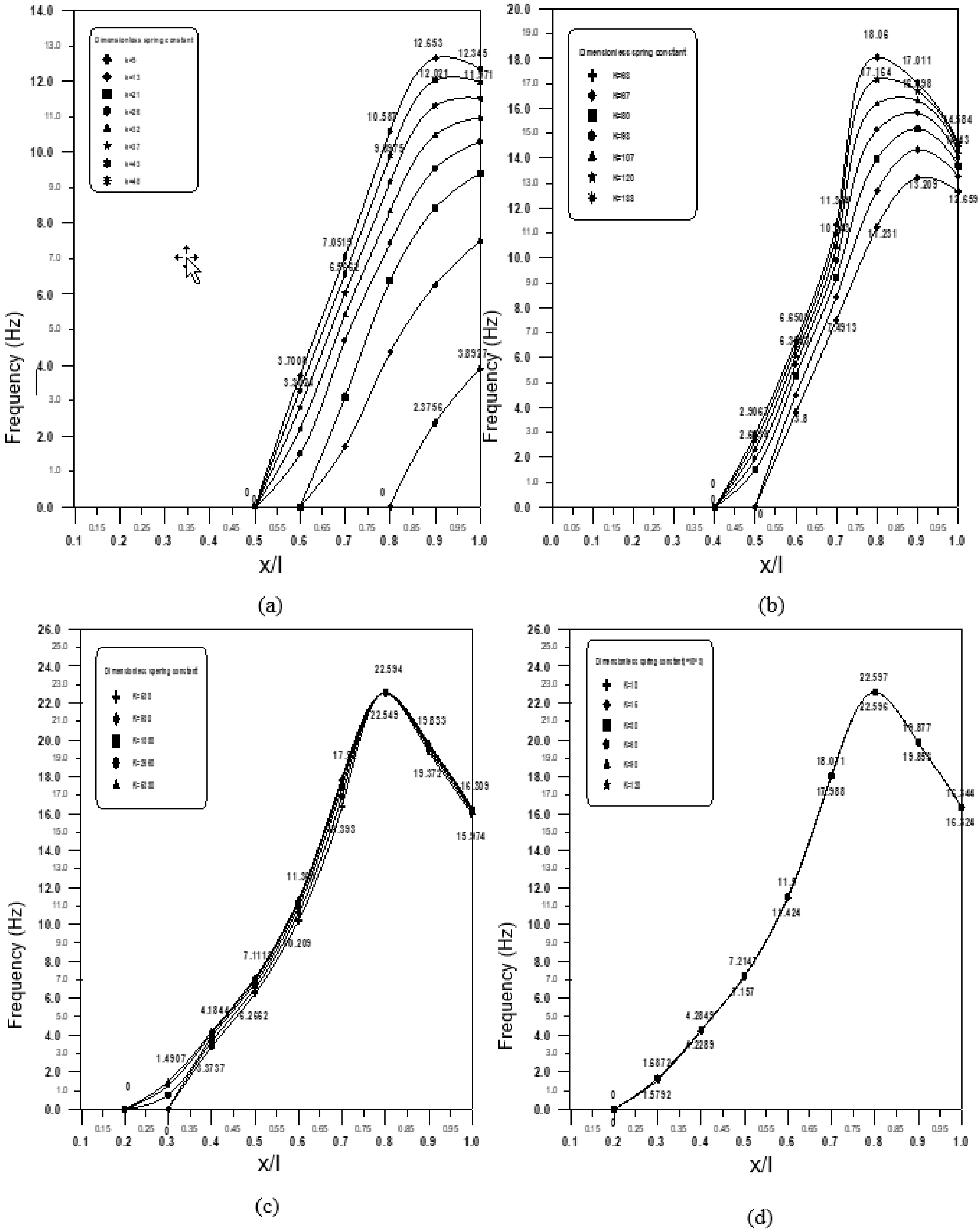
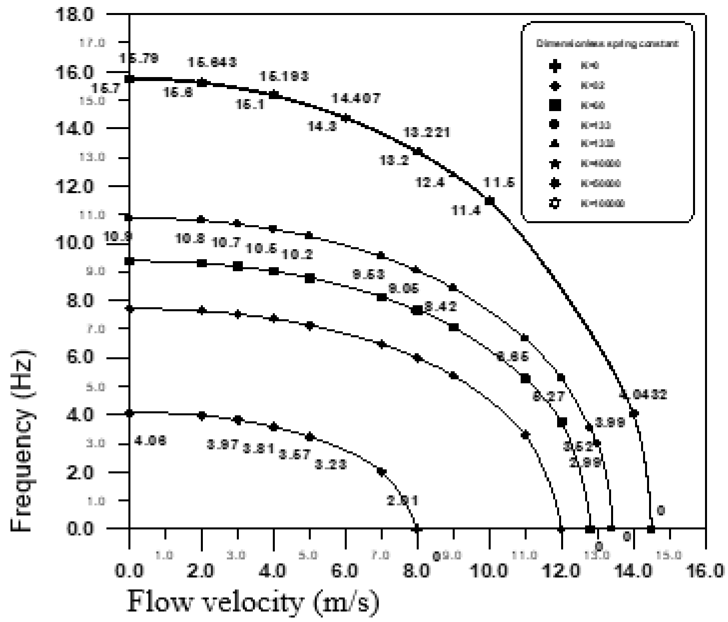
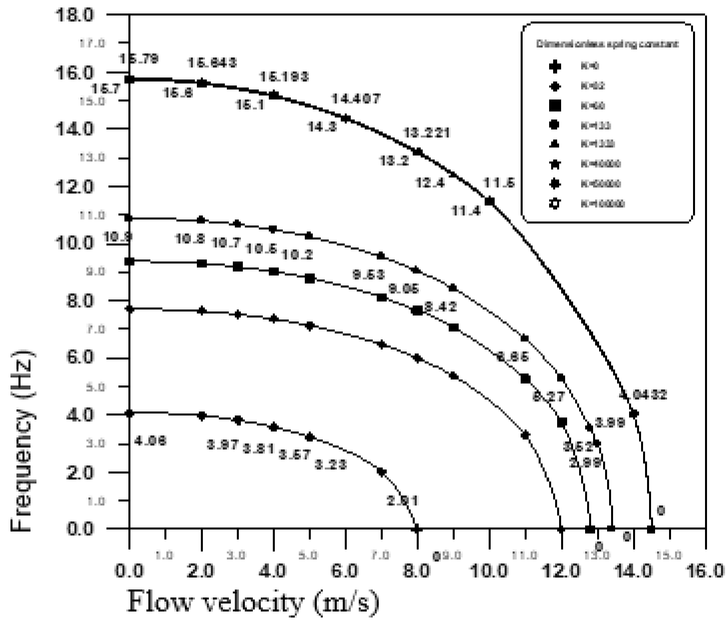


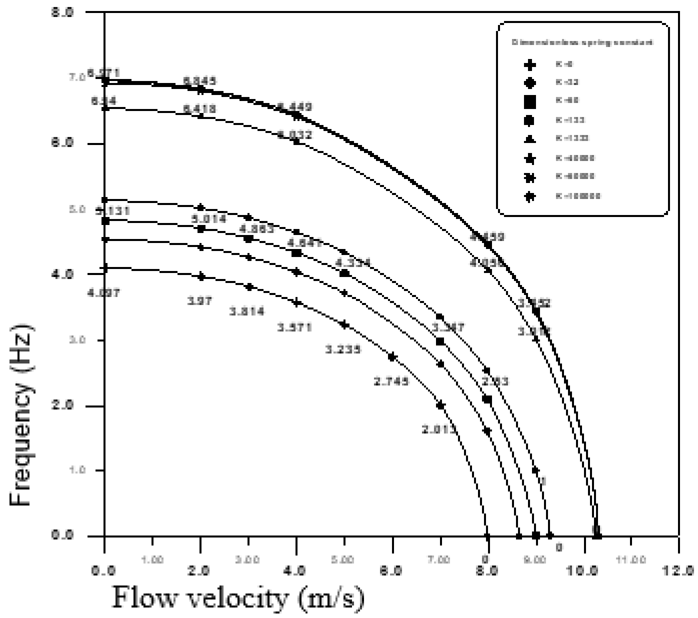

Fig.6. Presents effect of dimensionless spring constant and spring location on the system frequency with fluid velocity $U$=13 m/s.


Frequency (Hz)
Flow velocity (m/s)
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
16.0
18.0
15.79
15.643
15.193
14.407
13.221
11.5
15.7
15.6
15.1
14.3
13.2
12.4
11.4
10.9
10.8
10.7
10.5
10.2
9.53
9.05
8.42
6.65
6.27
4.0432
3.99
3.52
2.99
4.06
3.97
3.81
3.57
3.23
2.01
0

(a)


Frequency (Hz)
Flow velocity (m/s)
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
16.0
18.0
15.79
15.643
15.193
14.407
13.221
11.5
15.7
15.6
15.1
14.3
13.2
12.4
11.4
10.9
10.8
10.7
10.5
10.2
9.53
9.05
8.42
6.65
6.27
4.0432
3.99
3.52
2.99
4.06
3.97
3.81
3.57
3.23
2.01
0

(b)




(c )

Fig.7. Effect of flow velocity on the dimensionless frequency for different dimensionless spring constants and flow velocity.
(a) *U*=5m/s,(b) *U*=8m/s, (c) *U*=13m/s

Table3. Presents Effective Spring location for Different Values of Flow Velocity and Dimensionless Spring Constant.

| Dimensionless spring constant | Flow Velocity (m/s) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 6 | 7.975 | 11 | 13 | 17 | 20 | 24 | 25 |
| 2 | 0↔ 1 | 0.067↔ 1 | - | - | - | - | - | - |
| 4 | 0↔ 1 | 0.065↔ 1 | 0.82↔ 1 | 0.9↔ 1 | - | - | - | - |
| 6 | 0↔ 1 | 0.063↔ 1 | 0.75↔ 1 | 0.89↔ 1 | - | - | - | - |
| 8 | 0↔ 1 | 0.061↔ 1 | 0.7↔ 1 | 0.87↔ 1 | 0.93↔ 1 | - | - | - |
| 10 | 0↔ 1 | 0.06↔ 1 | 0.68↔ 1 | 0.858↔ 1 | 0.9↔ 1 | 0.95↔ 1 | - | - |
| 20 | 0↔ 1 | 0.058↔ 1 | 0.69↔ 1 | 0.854↔ 1 | 0.86↔ 1 | 0.94↔ 1 | - | - |
| 30 | 0↔ 1 | 0.056↔ 1 | 0.65↔ 1 | 0.6↔ 19 | 0.81↔ 1 | 0.9↔ 1 | - | - |
| 40 | 0↔ 1 | 0.054↔ 1 | 0.56↔ 1 | 0.64↔ 1 | 0.79↔ 1 | 0.88↔ 1 | - | - |
| 50 | 0↔ 1 | 0.053↔ 1 | 0.5↔ 1 | 0.62↔ 1 | 0.78↔ 1 | 0.86↔ 1 | 0.996↔ 1 | - |
| 60 | 0↔ 1 | 0.049↔ 1 | 0.48↔ 1 | 0.6↔ 1 | 0.77↔ 1 | 0.85↔ 1 | 0.99↔ 1 | - |
| 70 | 0↔ 1 | 0.047↔ 1 | 0.47↔ 1 | 0.59↔ 1 | 0.76↔ 1 | 0.845↔ 1 | 0.983↔ 1 | - |
| 80 | 0↔ 1 | 0.045↔ 1 | 0.465↔ 1 | 0.58↔ 1 | 0.758↔ 1 | 0.841↔ 1 | 0.98 ↔ 1 | - |
| 90 | 0↔ 1 | 0.043↔ 1 | 0.0461↔ 1 | 0.575↔ 1 | 0.75↔ 1 | 0.84↔ 1 | 0.977↔ 1 | - |
| 100 | 0↔ 1 | 0.041↔ 1 | 0.45↔ 1 | 0.5↔ 17 | 0.745↔ 1 | 0.836↔ 1 | 0.975↔ 1 | - |
| 400 | 0↔ 1 | 0.028↔ 1 | 0.398↔ 1 | 0.53↔ 1 | 0.74↔ 1 | 0.83↔ 1 | 0.964 ↔ 1 | - |
| 500 | 0↔ 1 | 0.026↔ 1 | 0.394↔ 1 | 0.528↔ 1 | 0.72↔ 1 | 0.82↔ 1 | 0.962↔ 1 | - |
| 600 | 0↔ 1 | 0.024↔ 1 | 0.389↔ 1 | 0.522 ↔ 1 | 0.71↔ 1 | 0.81↔ 1 | 0.961↔ 1 | - |
| 700 | 0↔ 1 | 0.022↔ 1 | 0.382↔ 1 | 051.↔1 | 0.708 ↔ 1 | 0.8↔ 1 | 0.96↔ 1 | - |
| 1000 | 0↔ 1 | 0.02↔ 1 | 0.381↔ 1 | 0.51↔ 1 | 0.70 ↔ 1 | 0.8↔ 1 | 0.96↔ 1 | - |

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